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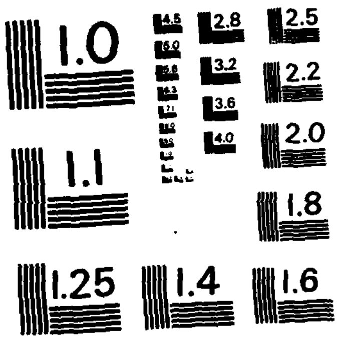
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A PRIMER ON ROUGH
SURFACE SCATTERING

SAI-83-140-WA
Contract N00014-82-C-2501

SCIENCE APPLICATIONS, INC.

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A PRIMER ON ROUGH SURFACE SCATTERING

SAI-83-140-WA

April 1983

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INTRODUCTION

This was originally intended as a brief introduction to the theory of waves scattered from a randomly rough surface. Now it is no longer so brief, but still introductory.

An alternate title to this report might be "Selections from Bass and Fuks". The majority of the material contained herein may be found in their excellent monograph "Wave Scattering from Statistically Rough Surfaces"[1]. I have rearranged some sections, to indicate their relationship and shortened others. The section on composite surfaces contains results due to Brown[2], which are not in Bass and Fuks, with a derivation due to Dashen[3].

I have attempted to include what I feel are the most important forms of the average field and average intensity for both large and small Rayleigh parameters. Then I have indicated, in the section on composite surfaces, one way to combine these forms to give a result which I hope is valid over a wide range of frequencies, angles, and surface statistics.

Shadowing corrections are included at the appropriate places, but no attempt to derive these results has been made. An excellent derivation due to Smith[4] is recommended to the reader. Some suggestions as to possible extensions of the use of shadowing are made, but their validity awaits additional verification.

It should be noted that this primer gives a single-bounce theory, describing a single encounter of a wave with a rough surface. The multiple-bounce theory, necessary for example in shallow water, is significantly harder. Attempts at a theory have been made, based on a Green's function for a waveguide with rough walls[5], but little comparison with data has occurred. A derivation of a loss per bounce using WKB theory has proven useful in computing loss due to the rough walls[6].

Currently at SAI a multiple-bounce theory from the moving ocean surface due to Dashen and Spofford is under development, with promising results thus far. If the current good results continue, it is very likely that this theory can be extended to a multiple-bounce theory in shallow water, including a rough bottom.

Following is a brief discussion of each section: Section I briefly summarizes what everyone should know about describing surfaces probabilistically. The reader should scan this, to fix notation at least. Various important relations are introduced here, for later reference.

Section II tries to relate Section I to reality, in a brief discussion of some of the rough surfaces of interest to an acoustician. A point made there perhaps deserves to be mentioned here also. The type of description of a random surface used in Section I certainly does not apply to all surfaces. It seems to work well for the ocean surface and it may be valid for the ocean basement. (Note that so little is known about the basement power spectra that with a little

ingenuity, one should be able to fit almost any basement scattering data. Talk about curve fitting!) However, this description is probably not valid for ice surfaces. People concerned with radar scattering may have to cope with an even wider variety of surfaces, buildings, tall grasses, trees, each of which may require a different description.

Section III attempts a brief discussion of the actual mechanisms of scattering. A little tolerance is perhaps called for on the reader's part, for the author is certainly no physicist. Even so, the rather simple-minded concepts in Section III have provided some insight to the author, and may prove helpful to the reader.

Section IV formulates basic solutions of the wave equation and Hemholtz' equation. Again, this is necessary to fix notation, define the appropriate Green's functions, reflection coefficients, etc.

Section V discusses scattering from a slightly rough surface, i.e., a small Rayleigh parameter. The solution given is the first order term in a series, where the Rayleigh parameter is the expansion parameter. The free surface is discussed first, followed by the rigid surface, and then a general interface. In each case, the mean field $\langle U \rangle = \bar{U}$ is discussed first, then the average intensity $\langle |U|^2 \rangle$. In discussing the intensity, a natural division occurs when the scattering surface is "small" or "large". Here "small" refers to a Fresnel zone on the surface, so that across the scattering surface the incident wave has little or no phase change. In the "large" case, where several Fresnel

zones are illuminated, the curvature of the wave across the surface must be taken into account. Typically, in the "small" case, a scattering kernel for a surface "patch" is derived, and this kernel is simply integrated over the illuminated surface in the "large" case. These distinctions do not arise for the mean field, because, as shown, the mean field propagates only along the specular path, relative to the mean (horizontal) plane.

Section VI discusses surfaces with a large Rayleigh parameter, and introduces the Kirchoff or tangent plane approximation. The author has attempted to make clear what assumptions, both physical and mathematical, are involved in this approximation. Perhaps it is worthwhile emphasizing here, as do Bass and Fuks, that this approximation is not the first term in some perturbation expansion, but simply an ad hoc approximation. Accordingly, no error estimate is possible, the only question can be "Does it work?". The solution does work in many cases, of course, but these remarks are worth keeping in mind.

In Section VI shadowing corrections are introduced, as seems appropriate for very rough surfaces. The scope and limitations of existing theory are indicated.

Composite surfaces are introduced in Section VII. Expressions for the average intensity are derived, which are simple, and, one hopes, valid for a wide range of Rayleigh parameters. Shadowing corrections are indicated as needed.

Finally Section VIII considers the power spectrum of a moving rough surface, specifically the ocean surface, rather briefly. The earlier results are extended to this case in a fairly straightforward manner.

A PRIMER ON ROUGH SURFACE SCATTERING

I. DESCRIPTION OF A RANDOM ROUGH SURFACE[7]

The simplest description of a random surface $z = \zeta\ (x,y,t)$ is obtained by assuming it is stationary in time and homogeneous in space, i.e., the probability of a certain height being exceeded is invariant under time and space translation, and the second order moments in surface height depend only on the differences of the space-time arguments. The first order probability density function for a stationary and homogeneous surface is independent of x,y and t. Thus

$$\mathrm{Prob}\ \left(\zeta(x,y,t) < z\right) = \int_{-\infty}^{z} w_1(u)\ du,$$

or

$$P\left(z < \zeta(x,y,t) < z+dz\right) \cong w_1(z)\ dz.$$

We are concerned with perturbations over a planar surface, (or a portion of a surface which may be regarded as planar, e.g., a portion of the lunar surface) and it is convenient to choose coordinates so the mean surface is at $z=0$.

In most of the applications involving scattering from rough surfaces, it is assumed w_1 is Gaussian or normal, i.e.,

$$w_1(z) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{\frac{-z^2}{2\sigma^2}} \qquad (1.1)$$

where σ is the standard deviation or r.m.s height of the surface around the mean value 0, $\sigma^2 = \langle\zeta^2\rangle$.

This assumption is quite often correct, in that the sea surface has a nearly Gaussian height distribution[8]. On other occasions, a Gaussian height distribution is assumed for want of a better. Here the central limit theorem, and the mathematical conveniences of normal distributions are two motivating factors for this assumption.

Two such mathematical conveniences are as follows:

1) Linear combinations of normal random variables are again normal. This means, for example, that a Gaussian surface may be written as the sum of two Gaussian surfaces.

2) Since differentiation also is a linear operation, it follows that the slope density function for the random variables $\gamma_x = \frac{\partial\zeta}{\partial x}$ and $\gamma_y = \frac{\partial\zeta}{\partial y}$ is also normal. For a fixed surface in a suitable coordinate system,

$$w_2(\gamma_x, \gamma_y) = \frac{1}{2\pi\Gamma_x\Gamma_y} \exp\left(-\frac{1}{2}\left(\frac{\gamma_x^{\ 2}}{\Gamma_x^{\ 2}} + \frac{\gamma_y^{\ 2}}{\Gamma_y^{\ 2}}\right)\right) , \qquad (1.2)$$

where $\Gamma_x^{\ 2} = \langle\frac{\partial\zeta^2}{\partial x}\rangle$, $\Gamma_y^{\ 2} = \langle\frac{\partial\zeta^2}{\partial y}\rangle$, ($\langle\ \rangle$ denotes expected value); and $\langle\frac{\partial\zeta}{\partial y}\frac{\partial\zeta}{\partial x}\rangle = 0$ in these coordinates. For a moving surface, $\langle(\frac{\partial\zeta^2}{\partial t})\rangle$, $\langle\frac{\partial\zeta}{\partial t}\frac{\partial\zeta}{\partial x}\rangle$ and $\langle\frac{\partial\zeta}{\partial t}\frac{\partial\zeta}{\partial y}\rangle$ also enter into the covariance matrix in the exponent.

Here the axes have been chosen to eliminate the spatial cross-term in the quadratic form in the exponent of w_2. Because of the special nature of the time coordinate, the space-time cross terms cannot be eliminated.

If the surface is isotropic, $\Gamma_x = \Gamma_y = \Gamma$, and

$$w_2(\gamma_x, \gamma_y) = \frac{1}{2\pi\Gamma^2} \exp\left(-\frac{1}{2\Gamma^2}(\gamma_x{}^2 + \gamma_y{}^2)\right). \tag{1.3}$$

An additional important descriptor of a random surface is the correlation function W, given by

$$W(x_1, y_1, t_1; x_2, y_2, t_2) = \frac{1}{\sigma^2} < \zeta(x_1, y_1, t_1)\zeta(x_2, y_2, t_2) > .$$

If the surface is homogeneous and stationary, then W depends only on the differences of the arguments above, i.e.,

$$W(x, y, \tau) = \frac{1}{\sigma^2} < \zeta(x_1, y_1, t)\zeta(x_1+x, y_1+y, t+\tau) >. \tag{1.4}$$

Note that in this definition, W is normalized so that $W(0,0,0) = 1$.

An additional important density function is the two point density function $w^{(2)}(z_1, z_2; r, \tau)$ defined by

$$\begin{aligned} &P(z_1 < \zeta(\vec{r}_1, t) < z_1 + dz_1, \text{ and} \\ &\quad z_2 < \zeta(\vec{r}_1 + \vec{r}, t+\tau) < z_2 + dz_2) \\ &\cong w^{(2)}(z_1, z_2; \vec{r}, \tau) dz_1 dz_2. \quad (\vec{r} = (x, y),\ \vec{r}_1 = (x_1, y_1)) \end{aligned}$$

For Gaussian surfaces, the following relation between $w^{(2)}$ and the correlation function W is valid:

$$w^{(2)}(z_1,z_2;\vec{r},\tau) = \frac{1}{2\pi\sigma^2 \sqrt{1-W^2(\vec{r},\tau)}} \cdot \qquad (1.5)$$

$$\exp\left[-\frac{z_1^2 - 2W(\vec{r},\tau)z_1z_2+z_2^2}{2\sigma^2\left(1-W^2(\vec{r},\tau)\right)}\right].$$

An alternative description of the random surface may be given in terms of the characteristic functions, which are the Fourier transforms of the height probability density functions:

$$f_1(k) = \int_{-\infty}^{+\infty} w_1(z)\, e^{ikz}\, dz = \langle e^{i\zeta(\vec{r},\tau)} \rangle, \qquad (1.6)$$

$$f_2(k_1,k_2;\vec{r},\tau) = \iint_{-\infty}^{+\infty} w^{(2)}(z_1,z_2;\vec{r},\tau)\, e^{i(k_1z_1+k_2z_2)}\, dz_1dz_2$$

$$(1.7)$$

$$= \langle e^{i\left(k_1\zeta(\vec{r}_1,t)+k_2\zeta(\vec{r}_1+\vec{r},t+\tau)\right)} \rangle.$$

It follows from the relation between $w^{(2)}$ and W that

$$\langle \zeta(\vec{r}_1,t)\ \zeta(\vec{r}_1+\vec{r},t+\tau) \rangle = \sigma^2 W(\vec{r},\tau) \qquad (1.8)$$

$$= -\left.\frac{\partial^2 f_2(k_1,k_2;r,\vec{\tau})}{\partial k_1 \partial k_2}\right|_{k_1=k_2=0}.$$

A further relation, to be used subsequently, is:

$$f_2(k_1,k_2;\vec{r},\tau) = \exp\left[\frac{-\sigma^2}{2}(k_1{}^2+k_2{}^2+2W(\vec{r},\tau)k_1k_2)\right] \quad (1.9)$$

(an identity valid for homogeneous and isotropic Gaussian surfaces).

The spectral density, or power spectrum $F(k_x,k_y,\omega)$ is given by

$$W(x,y,\tau) = \frac{1}{\sigma^2} \operatorname{Re} \iint_{-\infty}^{+\infty} dk_x dk_y \int_{-\infty}^{+\infty} d\omega F(k_x,k_y,\omega) e^{i(\vec{k}\cdot\vec{r}-\omega\tau)}, \quad (1.10)$$

$$\vec{k} = (k_x,k_y),\ \vec{r} = (x,y).$$

For a fixed (homogeneous) surface, since $W(\vec{r})=W(-\vec{r})$, it follows that $F(-\vec{k})=F(\vec{k})$. Furthermore F may be taken as real, and by the Wiener-Khinchine theorem, $F>0$. However, for a moving surface, F is not necessarily even in $\vec{k}$.

For the moving ocean surface, the first order dispersion relation for deep water gravity waves[9] implies that

$$F(\vec{k},\omega) = F_1(\vec{k})\ \delta(\omega-\omega_+), \quad (1.11)$$

where F_1 is the first order surface spectrum, and

$$\omega_+ = \operatorname{sgn}(k_x)\sqrt{gk}\ ,\ k^2=k_x{}^2+k_y{}^2,\ (g=9.81\ \text{m/sec}^2).$$

Even for the moving surface, $W(\vec{r},0)=W(-\vec{r},0)$, and similarly, $W(0,\tau)=W(0,-\tau)$. These symmetries imply, performing the ω-integration

$$W(\vec{r},\tau) = \frac{1}{\sigma^2} \iint_{-\infty}^{+\infty} dk_x dk_y \; F_1(\vec{k}) \cos(\vec{k}\cdot\vec{r}-\omega_+\tau). \quad (1.12)$$

For a fixed surface, the corresponding relation is

$$W(\vec{r}) = \frac{1}{\sigma^2} \iint_{-\infty}^{+\infty} dk_x dk_y \; F_1(\vec{k}) \cos(\vec{k}\cdot\vec{r}) \quad (1.13)$$

$$= \frac{1}{\sigma^2} \iint_{-\infty}^{+\infty} dk_x dk_y \; F_1(\vec{k}) \, e^{i\vec{k}\cdot\vec{r}} .$$

From the cosine transform of $W(\vec{r},\tau)$, one can also write directly

$$W(\vec{r},\tau) = \frac{1}{\sigma^2} \iint_{-\infty}^{+\infty} dk_x dk_y \; F(\vec{k},\omega) \, e^{i(\vec{k}\cdot\vec{r}-\omega\tau)}, \quad (1.14)$$

if $F(\vec{k},\omega)$ is now defined as

$$F(\vec{k},\omega) = \tfrac{1}{2}[F_1(\vec{k})\delta(\omega-\omega_+) + F_1(-\vec{k})\delta(\omega+\omega_+)] . \quad (1.15)$$

Both (1.11) and (1.15) are found in the literature[10].

Now let $\vec{k}=(k_x,k_y)=k(\cos\phi,\sin\phi)$, and $\vec{r}=(x,y) = r(\cos\theta,\sin\theta)$. Then (1.12) or (1.14) becomes

$$W(\vec{r},\tau) = \frac{1}{\sigma^2}\int_0^{\infty} dk \int_0^{2\pi} kF_1(k,\phi)\cos(kr\cos(\theta-\phi)-\omega_+\tau)d\phi, \quad (1.16)$$

and (1.13) becomes

$$W(\vec{r}) = \frac{1}{\sigma^2}\int_0^{\infty} dk \int_0^{2\pi} kF_1(k,\phi)\cos\big(kr\cos(\theta-\phi)\big)d\phi. \quad (1.17)$$

If the spectrum is separable, i.e., $F_1(k,\phi)= F_1(k)H(\phi)$, then the ϕ-integration may be possible. In particular, if the surface is isotropic, then $H\equiv 1$. Equations 1.16 and 1.17 reduce to

$$W(\vec{r},\tau) = \frac{2\pi}{\sigma^2}\int_0^{\infty} kF_1(k)J_o(kr)\ \cos(\omega_+\tau)dk \quad (1.18)$$

and

$$W(\vec{r})= \frac{2\pi}{\sigma^2}\int_0^{\infty} kF_1(k)J_o(kr)dk. \quad (1.19)$$

If the surface is cylindrical or one-dimensional, that is independent of y, then in (1.11) $F_1(\vec{k})= \delta(k_y)F_1(k_x)\delta(\omega-\omega_+)$, where now $\omega_+=\mathrm{sgn}(k_x)\sqrt{g|k_x|}$. From (1.12)

$$W(x,\tau) = \frac{1}{\sigma^2}\int_{-\infty}^{+\infty} F_1(k_x)\ \cos(xk_x-\omega_+\tau)dk_x, \quad (1.20)$$

and from (1.13),

$$W(x) = \frac{1}{\sigma^2} \int_{-\infty}^{+\infty} F_1(k_x)\cos(xk_x)dk_x. \qquad (1.21)$$

Just as $\sigma^2 = \langle\zeta^2\rangle$ is an important parameter in discussing the height distribution, the notion of a correlation time τ_0 and a correlation length ℓ_0 are important in discussing the correlation function.

Physically these describe a time (or length) interval over which different surface elements are correlated. On this scale the interaction of an incident wave with these elements should be considered coherent. Conversely, over several correlation times or distances, the surface elements are uncorrelated, and interaction with an incident wave is expected to be incoherent.

Naturally the transition from correlated to uncorrelated is fuzzy, as is the definition of correlation times or lengths. A local definition, emphasizing small-scale roughness, is given by

$$\nabla^2_{\vec{r}}\, W(0) = \frac{-1}{\ell_0{}^2}, \left(W''(0) = \frac{-1}{\ell_0{}^2}\right)^* , \frac{\partial^2 W(0)}{\partial\tau^2} = \frac{-1}{\tau_0{}^2} . \qquad (1.22)$$

A global definition, involving all roughness scales, is given by

$$\pi\ell_0^2 = \iint_{-\infty}^{+\infty} W(x,y)dxdy, \left(\ell_0 = \int_{-\infty}^{+\infty} W(x)dx.\right)^* \qquad (1.23)$$

* One dimensional form.

These varying definitions essentially coincide if W is a Gaussian correlation function, i.e., in one dimension

$$W(x,\tau) \cong \exp\left(\frac{-x^2}{\ell_o^{\,2}} - \frac{\tau^2}{\tau_o^{\,2}}\right). \tag{1.24}$$

For more general correlation functions, they need not agree. (Indeed, the local version may not exist, if W does not have second derivatives at 0.)

For a fixed surface, the dispersion of the derivatives $\frac{\partial\zeta}{\partial x}$, $\frac{\partial\zeta}{\partial y}$ is related to the height dispersion, and correlation lengths by

$$\Gamma_x^{\,2} = \sigma^2\left|\frac{\partial^2 W}{\partial x^2}(0,0)\right|, \quad \Gamma_y^{\,2} = \sigma^2\left|\frac{\partial^2 W}{\partial y^2}(0,0)\right|. \tag{1.25}$$

For a general surface, a knowledge of w_1 and W does not determine the higher-order statistics of the surface. However, for a Gaussian surface, a knowledge of σ^2 and W does completely specify all the higher moments of the surface, which is another motivation for preferring to deal with Gaussian surfaces.

A possible point of confusion should be mentioned here. When speaking of a Gaussian surface, what is meant is that the height probability density function is Gaussian (and therefore so is the slope density function). The correlation function for a Gaussian surface may or may not be Gaussian, (and the ocean surface, for example, does not have a Gaussian correlation function).

II. ACTUAL ROUGH SURFACES

For the moving ocean surface, the Gaussian assumption is generally valid. Further, there are many semi-empirical determinations of the power spectrum, (Pierson, Phillips, Pierson-Moskowitz)[11], so some degree of confidence is possible when describing the ocean surface statistically. See Tables 1 and 2 for typical parameter values.

When dealing with a planetary surface, the ocean basement, or lunar surface, the Gaussian density assumption need not be valid. Considerable disagreement exists as to an appropriate choice of power spectrum or correlation function.

Many of the scientists studying radar backscattering from the lunar surface assume a correlation function of the form[12]

$$W(x,y) = e^{-\alpha\sqrt{x^2+y^2}} \quad . \quad \left(W(x) = e^{-\alpha|x|}\right). \tag{2.1}$$

However, it has been shown that a correlation function for a reasonably smooth surface must have $\frac{\partial W}{\partial x}(0,0)=\frac{\partial W}{\partial y}(0,0)=0$, and also have second derivatives at (0,0)[13]. (The non-existence of $\nabla^2 W(0)$ implies that the surface has vertical faces.) Clearly the form for W given above fails to meet these requirements. Nevertheless, such a function is still being used, primarily because with the scattering model used, it is often possible to obtain a reasonably good fit to the data by a suitable choice of the parameter α.

It is quite possible that the scattering model used in lunar studies is inadequate, which perhaps explains

why the above correlation function is used to fit the data.

A kinder explanation, and perhaps a generally true statement, is that while near 0, the correlation function does have the required form (locally a Gaussian correlation function)

$$W(x,y) \cong 1 - \frac{1}{2}\left[\frac{\Gamma_x{}^2}{\sigma^2}x^2 + \frac{\Gamma_y{}^2}{\sigma^2}y^2\right], \tag{2.2}$$

at larger distances, W may decay like $e^{-\alpha|\vec{r}|}$, rather than $e^{-\alpha|\vec{r}|^2}$.

Physically this implies that for such a surface, the correlation length is determined by higher derivatives of the surface, curvature, etc., rather than depending only on r.m.s. height and slope, as does a Gaussian correlation function.

Using the relation between the power spectrum and W derived above, the correlation length using the first (derivative) definition can also be expressed in terms of the power spectrum, e.g., for a one-dimensional surface,

$$\frac{1}{\ell_o^2} = \frac{1}{\sigma^2}\int_{-\infty}^{+\infty} F_1(k_x)k_x{}^2\,dk_x, \tag{2.3}$$

and the theoretical requirement that W''(0) exist implies that $k_x{}^2\,F_1(k_x)$ must be integrable.

An easy way to meet this requirement in a model is to assume that there is some cut-off wave number k_c such that

$$F_1(k_x) = 0, \ |k_x|>k_c, \ (\text{or } F_1(\vec{r}) = 0, \ |\vec{r}|>k_c). \quad (2.4)$$

Such an assumption implies that there is some lower limit to the wave lengths present in a surface, or at least a lower limit to the wave lengths which effectively interact with an incident wave.

This is probably correct for the ocean surface. A popular power spectrum for gravity waves is the Phillips spectrum, which in one-dimension, $\cong k_x^{-3}$. But there is a lower limit on the wave length of gravity induced waves, below which capillary waves are present, and the power spectrum for capillary waves decays more rapidly than that for gravity waves. Capillary waves probably do not themselves interact significantly with an incident sound wave, but provide an energy transfer mechanism from the wind above the surface to the gravity waves.

A surface for which the integrability requirement on $k_x^2 F_1(k_x)$ is <u>not</u> met is that of ice[14]. The power spectrum for ice does apparently decay like k_x^{-3}, and this holds true on very small wave length scales. Accordingly, a notion such as correlation length has no meaning for an ice surface. Such a surface, (within the class of all surfaces), is on the boundary of fractals, or diffractals, and may well require somewhat different techniques to develop a scattering theory.

Ice surfaces, with many small fractures and nearly perpendicular faces, are quite different from planetary surfaces, where weathering presumably has rounded the edges, and filled in tiny fracture zones. Whether or not such

processes, in one form or another, have occurred for the lunar surface, or the ocean basement, is not clear.

There is no general consensus on a "typical" power spectrum for the ocean basement, and some oceanographers are reluctant to concede that such a notion has any meaning, due to the presence of sea mounts, significant trenches and ridges, etc. Opposed to this is the observation that data (e.g., propagation loss as a function of range and frequency) collected over several areas of the Pacific, (where a thin sediment presumably allows interaction with the ocean basement) exhibits a uniformity which suggests the same mechanisms are operating everywhere. If the scattering depends on the power spectrum of the ocean basement, then presumably there is a typical power spectrum.

An additional complication encountered when discussing bottom interaction in thin sediment areas of the ocean, such as the Pacific, is the water sediment interface. In regions where the sediment is on the order of 30 meters thick, apparently the sediment is "draped" over the rough basement, and follows a somewhat smoothed version of the basement contours.

So if it is necessary to treat the basement as a randomly rough surface, then presumably the sediment also is randomly rough. This introduces significant complications into a scattering model, as now a sound wave interacting with the basement before returning to the water column encounters three rough surfaces enroute, (with two transmission-reflections, and one reflection).

As the scattering theory to be described here is a single bounce theory, three encounters with rough surfaces

presents difficulties. Even if the present theory could be extended, and for example, three scattering kernels obtained, whose iteration presumably would represent the scattering effects, the numerical difficulties would in turn become formidable.

One possible way to avoid this difficulty is to model the sediment as a fluid, with a constant gradient in the sound speed, (and frequency dependent attenuation) but with an impedance match at the water sediment interface. Then the transmission coefficient is 1, and the distortion produced by refraction at the rough sediment surface can presumably be lumped into the scattering produced when the sound wave reaches the basement.

The result of this assumption is a single bounce theory, at least for incident waves approaching the sediment above a certain critical grazing angle. For waves with a grazing angle below this, the equivalent ray turns above the basement, and presumably the basement has little or no effect on such waves.

One should also note that if the model allowed an impedance discontinuity at the water sediment interface, many additional phenomena would be encountered. Among these could be a combination of parameters producing a critical angle below which near total reflection occurs. Such a critical angle is well-defined for planar interfaces. It is considerably more difficult to introduce such notions when the interface is rough, and the notion of "grazing angle" now must be interpreted in some average sense.

In thick sediment areas, such as the North Atlantic, with thicknesses ≅ 200 meters, it is very difficult for an incident wave to reach the basement, as almost all the rays turn above the basement. And as attenuation in the sediment is higher than in the water column, such waves are essentially damped before they can return to the water column. So the basement interaction is not significant.

Further, in thick sediment areas, the water sediment interface is gently undulating so that either the interface can be treated as planar, or perhaps as only a slightly rough surface. Much of the data from this area can be satisfactorly modeled without recourse to rough surface theory[15], so there is little need for a scattering model in thick sediment areas.

Of course, how one determines that a thick sediment model is no longer adequate, and a thin sediment scattering model allowing for interaction with a rough basement is needed, is not very clear. One can conceive of a sediment of an intermediate thickness, such that at longer ranges, the sound waves refract in the sediment, and turn above the basement, whereas at shorter ranges, a significant fraction of the sound energy in the sediment scatters off the basement.

III. DISCUSSION OF THE PHYSICS OF SCATTERING

Before proceeding to a discussion of the various mathematical models for scattering, it is useful to examine the physics involved.

Consider a portion of an incident sound wave scattering from a finite region of a rough surface. The portion of the incident wave might be a beam, (and therefore the beam pattern must be considered), or it could be part of a larger wave. The region considered is presumed to be small enough so that the incident wave may be regarded as plane, yet large enough so that averaging over the surface region gives meaningful and representative results.

If the incident wave ensonifies a large region of the surface, then the portion being considered here is part of the integrand of an integral over the large region. This final result is then obtained by summing or integrating the partial results.

Due to the randomness of the surface, such a sum is incoherent. To fix our ideas in the following discussion, consider the intensity of a plane wave scattered from a finite region of the surface.

There are two different contributions to the scattered field. One effect is the near-specular reflection around the specular path (between source and receiver) relative to the mean planar surface. Such an effect dominates near this specular path. This reflection, often described as "glints", is due to specular scattering off large planar facets of the surface. When looking down at a

sunlit ocean surface, the glints observed are from plane facets momentarily oriented so as to provide specular reflection from the sun to the eye. The strength of this component of the scattered field falls off rapidly away from the specular path.

A second component of the scattered field is diffuse diffraction due to the small scale roughness of the surface. This effect is not as strongly angle dependent is the first component described above, and accordingly tends to become the important component of the scattered field at scattering angles far from the specular angle.

"Tends" was used in the preceding sentence because as the surface becomes smooth, and approaches a plane, the diffuse component vanishes, while the "coherent component" described earlier reduces to the specular reflection from a plane. If, as suggested earlier, the theory is describing an integrand, to be integrated over some region, then this coherent component must reduce to a delta function within the integral. When the integral is evaluated, the delta function produces the desired specular reflection from the limiting plane surface.

Going to the other extreme, as the surface becomes rougher, with steeper slopes, then the specular component is weakened. This is because the planar facets are oriented over a larger range of angles, and accordingly fewer are properly oriented for specular reflection between source and receiver.

At the same time, for this increasingly rough surface, the diffuse component contains a larger fraction of

the total incident energy, and as the surface roughness increases, approaches a scattered field nearly independent of angle.

Some little sketches in Figure 1 borrowed from Beckman and Spizzichino[16] suggest this transition.

The first scattering component, or near specular component described above is often referred to in the acoustical literature as the "coherent field", while the second is termed the "incoherent field". Both contribute to the scattered average intensity, while only the coherent field contributes to the mean pressure field, which propagates along the specular path.

The acoustical distinction between the coherent and incoherent fields is perhaps not entirely clear, particularly when discussing the average intensity. However, there is a genuine difference between the two fields, because it is believed that different physical mechanisms give rise to them.

The experiments which provide the most convincing testimony to this involve radar backscattering from the moon. Using time delay to determine which annulus on the moon is illuminated by the radar pulse, and thereby determining the angle of incidence, the following results are obtained.

Electromagnetic theory predicts that when a circularly polarized wave is reflected from a plane, the reflected wave is also circularly polarized, but in the <u>opposite</u> sense. The radar return from the moon however has a

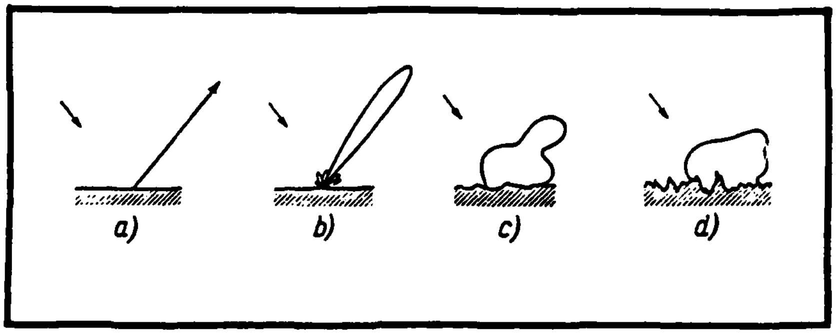

Figure 1. Transition from specular reflection to diffuse scattering. The surfaces are: (a) smooth, (b) slightly rough, (c) moderately rough, (d) very rough. (From Beckmann and Spizzichino)

component which is depolarized, that is, circularly polarized in the same sense as the transmitted wave. This depolarized component is the incoherent component or diffuse component, and the properly polarized component is the coherent component. A comparison between the power in the polarized component and the depolarized component is shown in Figure 2[17]. Observe the zero or minimum delay occurs at normal incidence, which for backscattering is the specular angle. Note that the angular dependence of the depolarized component is much weaker than the polarized, and exhibits no spike near the specular angle, as does the polarized component.

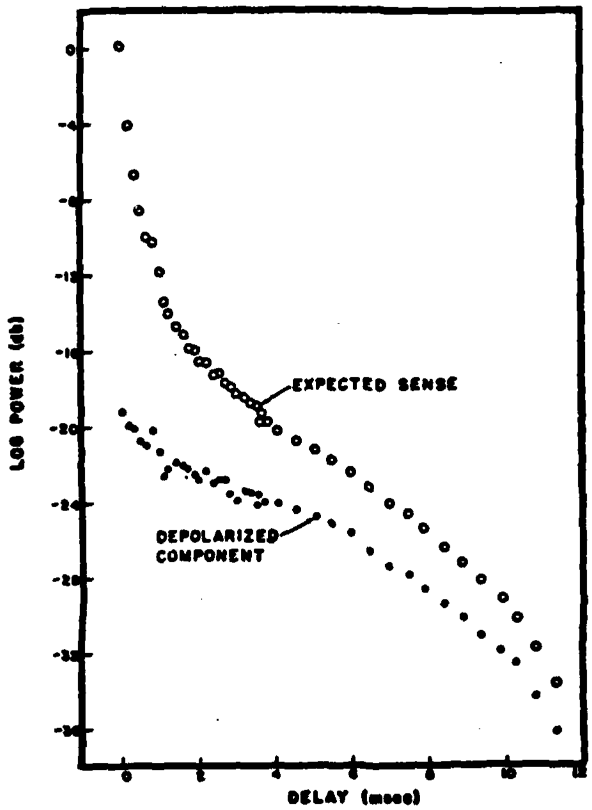

Figure 2. A comparison of the expected or polarized component $\overline{P}(t)$ of the echo power at 23 cm wavelength and the depolarized component $\overline{D}(t)$. Note the absence of an initial spike in the curve for $\overline{D}(t)$ corresponding to the quasi-specular scattering observed in $\overline{P}(t)$. (From Hagfors, 'Radar Astronomy')

IV. PROBLEM FORMULATION AND BASIC RELATIONS

Consider the wave equation for the potential U of a sound field,

$$\nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = -4\pi Q(R,t),$$

where $R = (x,y,z)$, c is the sound speed, and $4\pi Q$ is the source density distribution. The velocity v and the pressure p are given by

$$v = -\nabla U,\ p = \rho\frac{\partial U}{\partial t},$$

where ρ is the density of the medium.

We shall assume

$$Q(R,t)=Q(R)e^{-i\omega t},\ U(R,t)=U(R)e^{-i\omega t},$$

so that we obtain the Helmholtz equation

$$\nabla^2 U + k^2 U = -4\pi Q(R), \tag{4.1}$$

$k = \frac{\omega}{c}$.

For simplicity of exposition in the following, c will be assumed constant. Since the primary concern will be the interaction of an incident wave at the surface, relative to a distant source and receiver, corrections to be introduced due to refraction are fairly obvious. For example, a

term $\frac{1}{R_1}$ representing cylindrical spreading loss to a point on the surface should be replaced by the correct spreading loss along a suitable ray path from the source to the surface position; an incident angle θ_0 of a wave arriving at the surface should be related to the angle at the source that the corresponding ray makes with the vertical, the wave number $k=\omega/c$, uses the sound speed at the surface, etc.

The use of ray terminology already suggests that the frequencies involved are high enough that a ray description of the sound field makes sense, which gives a lower limit of 100 hz, or perhaps 50 hz.

The basic relation governing solutions of (4.1), derived using Green's formula, is

$$U(R) = \int_V Q(R')G(R,R')dR' + \frac{1}{4\pi} \int_\Sigma \left\{G(R,r)\frac{\partial U}{\partial n}(r) - U(r)\frac{\partial G}{\partial n}(R,r)\right\}dr, \tag{4.2}$$

where $R'\varepsilon V$, the volume containing the source Q and the observation point R, and $r\varepsilon\Sigma$, the boundary surface of V. $\frac{\partial}{\partial n}$ denotes differentiation with respect to the exterior normal to the surface Σ. The function G is the Green's function, a solution of (4.1) when $Q=\delta(R-R')$, δ the Dirac delta function.

Considerable simplification of (4.2) can be achieved by a suitable choice of the Green's function. Suppose $\sum$ is a plane surface. Then two convenient choices are

$$G_\pm(R,R') = \frac{e^{ik|R-R'|}}{|R-R'|} \pm \frac{e^{ik|R_1-R'|}}{|R_1-R'|} \tag{4.3}$$

where R_1 is the point obtained by reflecting R with respect to the plane $\sum$, (if $\sum$ is the plane z=0, then $R_1 = (x,y,-z)$).

Then it results that

$$\left.\frac{\partial G_+}{\partial n}\right|_{\sum} = 0, \quad G_-\Big|_{\sum} = 0, \tag{4.4}$$

(for $\sum = \{z=0\}$, $\frac{\partial}{\partial n} = \frac{-\partial}{\partial z}$).

Using either G_+ or G_-, (4.2) becomes

$$U(R) = \int_V Q(R')G_+(R,R')dR' + \frac{1}{4\pi}\int_{\sum} G_+(R,r)\frac{\partial U}{\partial n}(r)dr, \tag{4.5}$$

or

$$U(R) = \int_V Q(R')G_-(R,R')dR' - \frac{1}{4\pi}\int_{\sum} U(r)\frac{\partial G_-}{\partial n}(R,r)dr. \tag{4.6}$$

If the source function Q(R) is zero outside a sphere of radius L, and if

$$\frac{L}{|R|}, \ \frac{kL^2}{|R|} << 1, \tag{4.7}$$

then the source term in the expressions (4.2, 4.5, 4.6) satisfies

$$\int_V Q(R') \frac{e^{ik|R-R'|}}{|R-R'|}dR' = d(K)\frac{e^{ik|R|}}{|R|}, \tag{4.8}$$

where $d(K) = \int_V Q(R')e^{-iK\cdot R'}dR'$, $K = \frac{kR}{|R|}$.

The region of space where the inequalities (4.7) are satisified is called the Fraunhofer zone relative to the source. Equation (4.8) states that at sufficiently large distances, an extended source looks like a point source.

One can verify directly that a solution (4.1) with $Q(R)=0$ is given by a plane wave, of the form

$$U(R) = Ae^{iq \cdot R}, \tag{4.9}$$

where $q = (q_x, q_y, q_z)$, and $|q|^2 = k^2$.

One of the useful properties of plane waves is that a spherical wave, as in (4.8), can be represented in terms of plane waves. In fact

$$\frac{e^{ik|R|}}{|R|} = \frac{i}{2\pi} \int_{-\infty}^{+\infty}\int e^{i(xq_x+yq_y+|z|q_z)} \frac{dq_x dq_y}{q_z} \tag{4.10}$$

where $q_z = \sqrt{k^2-q_x{}^2-q_y{}^2}$. So when discussing a wave incident upon a surface, if only a restricted region of the surface is considered, the incident wave may be usefully modeled as a plane wave.

For reference purposes, let us describe the propagation of a plane wave in an infinite space, consisting of two half-spaces, separated by the plane $z=0$, with density and sound speeds ρ, c, for $z>0$, and ρ_2, c_2 for $z<0$. Let U, U_2 denote the corresponding potentials.

Continuity conditions at the interface lead to the boundary conditions

$$\frac{\partial U}{\partial n} = \frac{\partial U_2}{\partial n}, \ \rho U = \rho_2 U_2 \tag{4.11}$$

(where $\frac{\partial}{\partial n} = \frac{-\partial}{\partial z}$, as before).

Let $k = \omega/R$, $k_2 = \omega/c_2$, $K = (k_x, k_y, k_z) = k(\sin\theta\cos\phi, \sin\theta\sin\phi, -\cos\phi)$, $K' = (k_x, k_y, -k_z)$, $K_2 = (k_2(\sin\theta_2\cos\phi, \sin\theta_2\sin\phi, -\cos\theta_2)$, where θ, θ_2 are the incident angles of the incident and transmitted waves respectively, and ϕ the azimuthal angle, relative to some fixed vertical plane.

K, K', and K_2 are the wave vectors of the incident reflected and transmitted waves, respectively (see Figure 3). Of course, all these vectors lie in the same vertical plane containing the source and the receiver at R, defined by the azimuthal angle ϕ.

Snell's law defines θ_2 in terms of θ, namely $\frac{\sin\theta_2}{c_2} = \frac{\sin\theta}{c}$.

If the incident wave is $U_0 = Ae^{iK \cdot R}$, then the reflected wave is

$$U_r = V(\theta)Ae^{iK' \cdot R}, \tag{4.12}$$

and the transmitted wave is

$$U_T = W(\theta)Ae^{iK_2 \cdot R}. \tag{4.13}$$

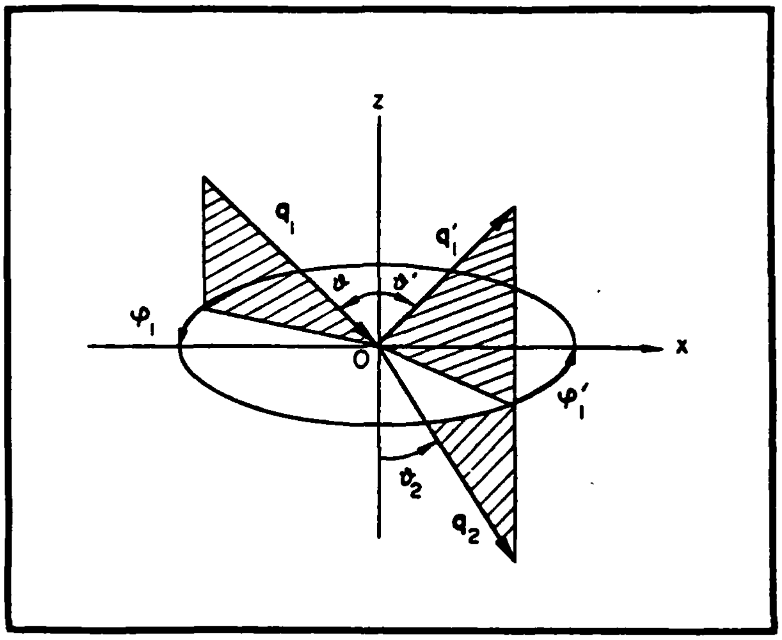

Figure 3

The field in the upper half-space is given by $U_0 + U_r$.

$V(\theta)$, $W(\theta)$ are given by

$$V = \frac{(\rho_2/\rho)\cos\theta - \sqrt{(c/c_2)^2 - \sin^2\theta}}{(\rho_2/\rho)\cos\theta + \sqrt{(c/c_2)^2 - \sin^2\theta}}, \tag{4.14}$$

$$W = \frac{2\cos\theta}{(\rho_2/\rho)\cos\theta + \sqrt{(c/c_2)^2 - \sin^2\theta}}. \tag{4.15}$$

Let $\eta = (\rho c)/(\rho_2 c_2)$ denote the surface impedance (the ratio of the acoustic impedance of the lower half space to the upper). Then a perfectly free surface $(\rho_2 c_2 \to 0)$ corresponds to $\eta=\infty$, and a perfectly rigid surface $(\rho_2 c_2 \to \infty)$ corresponds to $\eta=0$.

The boundary conditions (4.11) for a perfectly free surface $(\rho_2=0)$ reduce to

$$U = 0, \tag{4.16}$$

and the corresponding reflection coefficient $V = -1$. (G_- satisfies this condition.)

For a perfectly rigid surface, the boundary conditions are

$$\frac{\partial U}{\partial n} = 0, \tag{4.17}$$

and the corresponding reflection coefficient $V = +1$. (G_+ satisfies this condition.)

V. PERTURBATION SOLUTION FOR SLIGHTLY ROUGH SURFACES

A. Perfectly Free Surface[18]

Suppose the rough surface $z = \zeta(r)$, $r = (x,y)$ is such that the boundary condition $U(r,\zeta(r)) = 0$ can be approximated by

$$U(r,0) + \zeta(r) \frac{\partial U}{\partial z}(r,0) = 0 \tag{5.1}$$

Now represent U as the sum of a mean field and a fluctuating field, $U = \overline{U} + u$, where $\langle U \rangle = \overline{U}$, $\langle u \rangle = 0$. Also assume $\langle \zeta \rangle = 0$.

Then averaging (5.1) and subtracting the resulting equation from (5.1) results in

$$u(r,0) + \frac{\partial \overline{U}(r,0)}{\partial z} \zeta(r) = 0 \tag{5.2}$$

Subtracting (5.2) from (5.1) and averaging gives

$$\overline{U}(r,0) + \langle \frac{\partial u}{\partial z}(r,0)\zeta(r) \rangle = 0 \tag{5.3}$$

In deriving (5.2), the term $\frac{\partial u}{\partial z}\zeta - \langle \frac{\partial u}{\partial z}\zeta \rangle$ was neglected. A sufficient condition for this (i.e., for $|u| \gg \left|\frac{\partial u}{\partial z}\zeta\right|$, etc.), is

$$\sigma k \sqrt{\left|1 - \left(\sin\theta + \frac{2\pi}{K\lambda}\right)^2\right|} \ll 1 \tag{5.4}$$

Here θ is the angle of incidence, k is the acoustic wave number, σ is the r.m.s. surface height, $\sigma^2 = \langle\zeta^2\rangle$, and ℓ is the correlation length of the surface, as in (1.22).

For $k\ell >> 2\pi$, this reduces to

$$\sigma k \cos\theta << 1 \tag{5.5}$$

For $k\ell << 2\pi$, this reduces to

$$\sigma k \frac{2\pi}{k\ell} = 2\pi\frac{\sigma}{\ell} = 2\pi\Gamma << 1 \tag{5.6}$$

where $\Gamma = \frac{\sigma}{\ell}$ = r.m.s. slope of ζ.

If $\sin\theta \cong 1$, (near grazing) this reduces to

$$\sigma k \sqrt{\frac{2\pi}{k\ell}} \cong \sigma \sqrt{\frac{2\pi k}{\ell}} << 1 \tag{5.7}$$

The parameter $2\sigma k\cos\theta$ introduced in 5.5 is the Rayleigh parameter.

a) The Mean Field

Using the Green's function G_- in (4.4) for the plane $z=0$, the boundary condition (5.3) results is $(R = (x,y,z),\ r' = (x',y') = (x',y',0))$,

$$u(R) = \frac{1}{2\pi}\frac{\partial}{\partial z}\int_{z'=0} \frac{e^{ik|R-r'|}}{|R-r'|}\frac{\partial\bar{U}}{\partial z'}(r')\zeta(r')dr' \tag{5.8}$$

Taking the z-derivative of (5.8), multiplying by $\zeta(r)$, and averaging, the boundary condition (5.2) gives the equation

$$\overline{U}(r) = \frac{-\sigma^2}{2\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \int_{z'=0} \frac{e^{ik\sqrt{|\rho|^2+z^2}}}{\sqrt{|\rho|^2+z^2}} W(\rho)\frac{\partial \overline{U}}{\partial z'}(r+\rho)d\rho, \qquad (5.9)$$

where $r' = r+\rho$.

Assume the incident field is a plane wave, so that the mean field has the form

$$\overline{U}(r,z) = A\{e^{iK\cdot R} + Ve^{iK'\cdot R}\}, \qquad (5.10)$$

where K, K' are as defined in Section IV. (The fact that the reflected wave is directed along K' results from inserting (5.10) (with K' replaced by an abitrary vector) into (5.9). That is, the mean field is directed along the specular path, which is a result generally true.)

Using (5.10) to express $\overline{U}$ in (5.9), the resulting equation can be solved for the reflection coefficient V, resulting in the expression

$$V = -1 + 2\cos\theta\,\eta_\mu(\theta,\phi), \qquad (5.11)$$

where the "effective admittance" η_μ of the rough surface is defined by

$$\eta_\mu(\theta,\phi) = \frac{ik\sigma^2}{2\pi} \lim_{z\to 0} \frac{\partial^2}{\partial z^2} \int_{-\infty}^{+\infty}\int \frac{e^{ik\sqrt{|\rho|^2+z^2}}}{\sqrt{|\rho|^2+z^2}} W(\rho)e^{ik_\perp\cdot\rho}d\rho \qquad (5.12)$$

where $k_\perp = (k_x, k_y)$.

To evaluate (5.12) for a general $W(\rho)$ would require detailed knowledge of W. But if W depends only on $|\rho|$, i.e., the surface is assumed isotropic, one of the integrations in (5.12) can be performed, the z differentiation and limit taken, with the result that

$$\eta_\mu(\theta) = k^2\sigma^2 - ik\sigma^2 \int_0^\infty \frac{e^{iky}}{y} \frac{d}{dy} \left[W(y)J_0(ky\sin\theta)\right]dy \qquad (5.13)$$

(The derivation of (5.13) depends in an essential manner on $W'(0) = 0$. The reader should recall the discussion on smooth surfaces, and the corresponding requirements on W.)

If now the correlation length ℓ is introduced as a scale factor, $W(y) = \tilde{W}(\frac{y}{\ell})$, various limiting forms of (5.13) are derived by Bass:

For $k\ell \ll 1$, or equivalently $2\pi\ell \ll \lambda$,

$$\eta_\mu \cong k^2\sigma^2 - \frac{ik\sigma^2}{\ell} \int_0^\infty \frac{1}{x} \frac{d\tilde{W}(x)}{dx}dx \qquad (5.14)$$

For $k\ell \gg 1$, and $(\frac{\pi}{2} - \theta) \gg \frac{1}{\sqrt{k\ell}}$,

$$\eta_\mu \cong k^2\sigma^2\cos\theta, \ (\psi = \frac{\pi}{2} - \theta \text{ is the grazing angle}) \qquad (5.15)$$

or, recalling 5.11,

$$V(\theta) = -1 + 2k^2\sigma^2\cos^2\theta. \qquad (5.16)$$

Finally, for $k\ell \gg 1$, and $k\ell\phi^2 \ll 1$, (grazing incidence),

$$\eta_\mu \cong \frac{2k^2\sigma^2}{\sqrt{2\pi k\ell}} e^{-3\pi i/4} \int_0^\infty \frac{1}{\sqrt{x}} \frac{d\tilde{W}}{dx}(x)dx \qquad (5.17)$$

(5.16) will occur subsequently as a limiting form in Section VI for surfaces with a small Rayleigh parameter.

b) The Intensity of the Fluctuating Field for a Free Surface

Begin with the formula (5.8) derived earlier for the fluctuating component of the field, u(R), except now restrict the integral to the actual scattering surface S ensonified by the incident wave, so

$$u(r) = \frac{1}{2\pi} \frac{\partial}{\partial z} \int_S \frac{e^{ik|R-r'|}}{|R-r'|} \frac{\partial \bar{U}}{\partial z'}(r')\zeta(r')dr' \qquad (5.18)$$

It is convenient to place the source at a height z_0 above the surface, at $(0,0,z_0)$, and the receiver at (D,z), see Figure 4. In Figure 4, $\vec{R}_1$, $\vec{R}_2$ denote vectors connecting an arbitrary point r of S to the source and receiver respectively, (from the source, to the receiver). Then

$$R_1 = |\vec{R}_1| = \sqrt{z_0{}^2+|r|^2},\ R_2 = |\vec{R}| = \sqrt{z^2+|D-r|^2} \qquad (5.19)$$

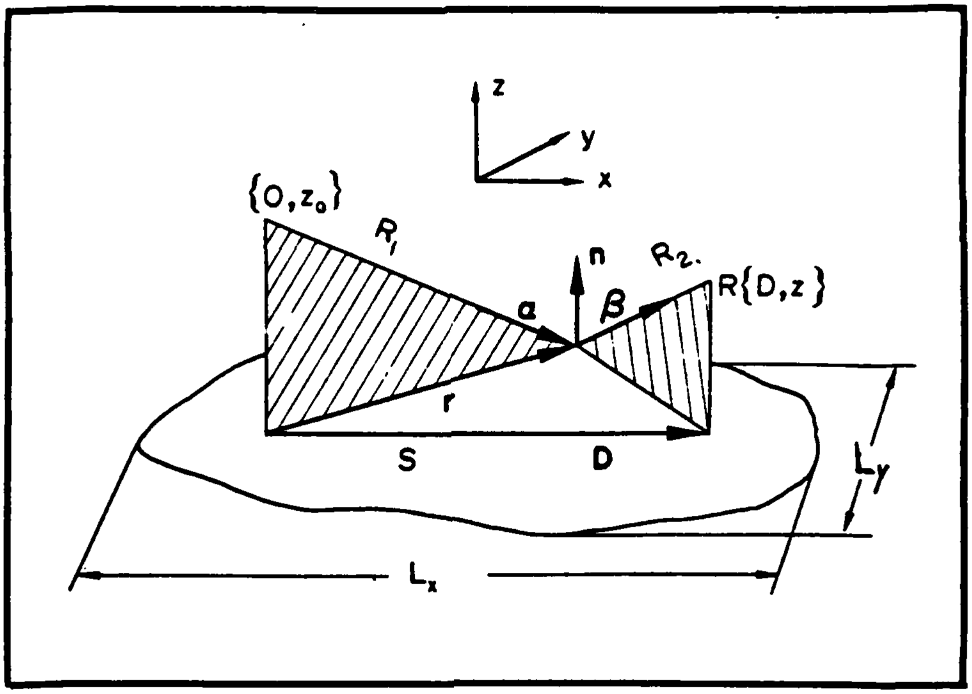

Figure 4

Then

$$\alpha = \frac{\vec{R}_1}{R_1} = \left(\frac{r}{R_1}, \frac{-z_0}{R_1}\right), \ \beta = \frac{\vec{R}_2}{R_2} = \left(\frac{D-r}{R_2}, \frac{z}{R_2}\right) \tag{5.20}$$

are the corresponding unit vectors.

From (5.11) it follows that the reflection coefficient V differs from -1 by an amount which is second order in the perturbation parameter (e.g., the Rayleigh parameter). Accordingly $\overline{U}(R)$ in (5.17) may be replaced by the planar result for a perfectly free surface, for a point source at $(0,0,z_0)$ given in (4.3), which in the present coordinate system has the form

$$U_0(R) = \frac{e^{ik|R-S|}}{|R-S|} - \frac{e^{ik|R-S'|}}{|R-S'|} \tag{5.21}$$

where $S = (0,0,z_0)$ is the location of the source, and $S' = (0,0,-z_0)$ is the image source.

Then $\frac{\partial U_0}{\partial z}(r,0) = -2ikz_0 \frac{e^{ikR_1}}{R_1{}^2}$, for $kz_0 \gg 1$. Similarly, for $kz \gg 1$, taking the z-derivative in (5.18) gives

$$u(R) = \frac{k^2}{\pi} \int_S \frac{e^{ik(R_1+R_2)}z_0 z}{R1^2 R_2{}^2} \zeta(r)dr. \tag{5.22}$$

(5.21) immediately implies

$$\langle |u(R)|^2 \rangle = \frac{k^4 z_0^2 z^2}{\pi^2} \iint_{S\times S} \frac{e^{ik(R_1+R_2-R_1'-R_2')}}{R_1^2 R_2^2 R_1'^2 R_2'^2} \langle \zeta(r)\zeta(r') \rangle dr\, dr', \tag{5.23}$$

where R_1', R_2' are the norms of the vectors connecting the point r' in S to source and receiver respectively.

Let $\rho = r'-r$ be a new variable. Then $\langle \zeta(r)\zeta(r') \rangle = \sigma^2 W(\rho)$. Now if the dimensions of S are greater than ℓ_x, ℓ_y, (the correlation lengths of ζ), the ρ integration may be extended to infinity, ($W(\rho) \sim 0$ for $|\rho| \gg \ell_x, \ell_y$). Further, if $\min\left(\ell, \sqrt{\frac{R}{k}}\right) \ll R_1, R_2$, then $R_1'^2 \cong R_1^2$, $R_2'^2 \cong R_2^2$. Finally, expanding the exponent in terms of ρ,

$$R_1 + R_2 - R_1' - R_2' = \left[\frac{D-r}{R_2} - \frac{r}{R_1}\right] \cdot \rho \tag{5.24}$$

if

$$k\ell_x^2 \cos^2\theta,\ k\ell_y^2 \ll M \tag{5.25}$$

where $M = 2R_1R_2/(R_1+R_2)$. Note that $\sqrt{M/k\cos^2\theta}$ and $\sqrt{M/k}$ are the (x-y) dimensions of the Fresnel zone relative to the source and receiver. So (5.25) implies that the field is considered only in the Fraunhofer zone relative to irregularities of dimension ℓ_x, ℓ_y.

Collecting all the above, (5.22) reduces to

$$<|u(R)|^2> = \frac{k^4\sigma^2}{\pi^2} \int_S \frac{\alpha_z{}^2\beta_z{}^2}{R_1{}^2R_2{}^2}\, dr \int_{-\infty}^{+\infty} W(\rho)e^{-ik(\beta_\perp-\alpha_\perp)\cdot\rho}d\rho \qquad (5.26)$$

(Recall that $\alpha = \frac{\vec{R}_1}{R_1} = (\frac{r}{R_1}, \frac{-z_0}{R_1}) = (\alpha_\perp, \alpha_z)$, and $\beta = \frac{\vec{R}_2}{R_2} = (\frac{D-r}{R_2}, \frac{z}{R_2}) = (\beta_\perp, \beta_z)$.)

The consideration of various limiting cases is useful in obtaining a deeper understanding of (5.26). Before introducing these, recall the assumptions made in deriving (5.26), namely (5.25), and the inequalities

$$L >> \ell = \max(\ell_x,\ell_y);\ \min(\ell,\sqrt{\frac{R}{k}}) << R_1,\ R_2; \qquad (5.27)$$

where L is a characteristic dimension of the scattering region S.

Now suppose $k\ell >> 1$, large-scale irregularities. Then, in (5.26)

$$\int_{-\infty}^{+\infty} W(\rho)e^{-ik(\beta_\perp-\alpha_\perp)\cdot\rho}\, d\rho \cong (\frac{2\pi}{k})^2\ \delta(\beta_\perp-\alpha_\perp) = (\frac{2\pi}{k})^2\ \delta(\frac{D-r}{R_2} - \frac{r}{R_1}), \qquad (5.28)$$

recalling (5.20).

Note that $\alpha_\perp = \beta_\perp$ defines the point $r = r_0$ of specular reflection in S relative to the source and receiver, (see Figure 5).

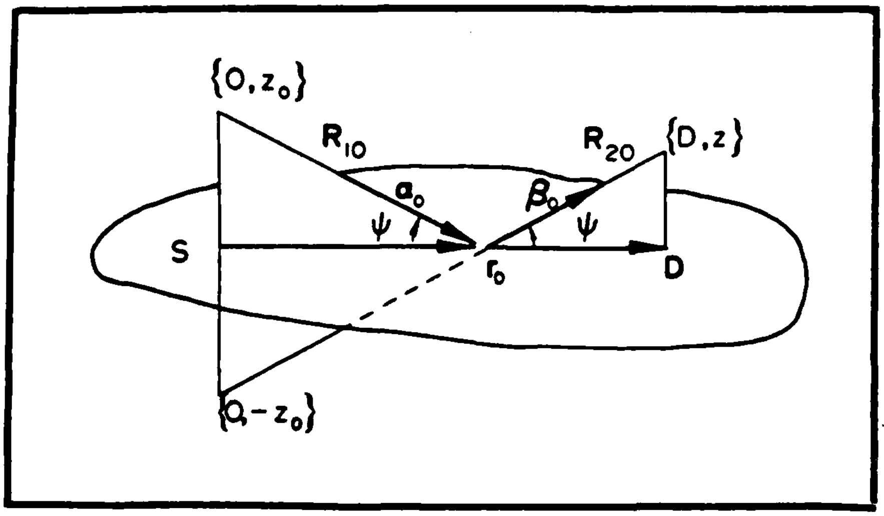

Figure 5

In the opposite case, $k\ell << 1$, (fine irregularities), the exponent in the ρ-integral is small for all angles, and

$$\int_{-\infty}^{+\infty} W(\rho)e^{-ik(\beta_\perp-\alpha_\perp)\cdot\rho}d\rho \cong \int_{-\infty}^{+\infty} W(\rho)d\rho = \ell_x\ell_y \int_{-\infty}^{+\infty} \tilde{W}(\xi,\eta)d\xi d\eta$$

$$= C\ell_x\ell_y \qquad (5.29)$$

where C is constant, approximately one.

Here the scale factors ℓ_x, ℓ_y have been introduced, so that $W(x,y) = \tilde{W}(\frac{x}{\ell_x}, \frac{y}{\ell_y}) = \tilde{W}(\xi,\eta)$.

i) "Small" Scattering Surface S

Suppose in addition to (5.25) and (5.27),

$$k\ell L \cos^2\theta,\ L << 2R_1R_2/(R_1+R_2). \qquad (5.30)$$

(These inequalities are somewhat weaker than requiring that the entire surface S be in the Fraunhofer zone relative to the receiver. Bass refers to the region described by (5.30) as the spectral partition zone.)

In this case, the r-dependence of the integrand in (5.26) becomes so weak that the r-integrand may be approximated by evaluating the integrand at an arbitrary point in S, and multiplying by $|S|$, the area of S.

In this case, it is convenient to reinterpret $|u(R)|^2$, and rather than think in terms of a path connecting source and receiver, regard the expression in (5.26) as a function of the incoming and outgoing wave vectors, in the directions given by α,β, respectively. Here we regard α,β as directed to the center of the small scattering surface S. Denoting $<|u(R)|^2>$ by $J(\alpha,\beta)$,

the result is

$$J(\alpha,\beta) = \frac{4k^4}{R_1{}^2R_2{}^2} \left|S\right| \alpha_z{}^2\beta_z{}^2F_1(k(\beta_\perp-\alpha_\perp)). \qquad (5.31)$$

F_1 is the spatial power spectrum of the surface, that is, (see 1.13),

$$F_1(K) = \frac{\sigma^2}{(2\pi)^2} \int\int_{-\infty}^{+\infty} W(\rho)e^{iK\cdot\rho}d\rho \qquad (5.32)$$

The appearance of the vector $k(\beta_\perp-\alpha_\perp)$ as the argument of F_1 in 5.31 implies that in first order perturbation theory, the average intensity of the scattered field depends upon only one component of the surface spectrum. This is a type of spatial resonance.

In the special case $k\ell >> 1$, estimating (5.32) by (5.28), (5.31) reduces to

$$J(\alpha,\beta) = \frac{4k^2\sigma^2}{R_1{}^2R_2{}^2} \left|S\right| \alpha_z{}^2\beta_z{}^2\delta(\beta_\perp-\alpha_\perp) \qquad (5.33)$$

so the long irregularities in the surface scatter only in the specular direction, $\alpha_\perp = \beta_\perp$.

Conversely, if $k\ell << 1$, from (5.29), the F_1-term in (5.31) $\sim \sigma^2\ell^2$. In this case the scattering is diffuse, and is proportional to $(k\sigma)^2 \cdot (k\ell)^2$.

ii) "Large" Scattering Surface S

Now suppose that the ensonified region S is so large that some portion of S lies outside the spectral partition zone, that is, (5.30) is not satisfied.

In this case, in (5.26), the r-integral over S must be evaluated. For reference purposes, (5.26) may be rewritten in terms of the power spectrum F_1 as

$$<|u(R)|^2> = 4k^4 \int_S \frac{\alpha_z{}^2\beta_z{}^2 F_1(k(\beta_\perp - \alpha_\perp))}{R_1{}^2 R_2{}^2}\, dr \qquad (5.34)$$

A generalization of this result, for composite surfaces, will be encountered subsequently.

Suppose $k\ell >> 1$, the case of large irregularities. Then, using the estimate (5.28),

$$<|u(R)|^2> = 4k^2\sigma^2 \int_S \frac{\alpha_z{}^2\beta_z{}^2}{R_1{}^2 R_2{}^2}\, \delta\left(\frac{D-r}{R_2} - \frac{r}{R_1}\right) dr$$

Changing coordinates in the integral, (see Appendix A for details), gives the result

$$<|u(R)|^2> = \frac{4k^2\sigma^2\alpha^2{}_{0z}\beta_{0z}{}^2}{\beta_{0z}{}^2(R_{10}+R_{20})^2} = \frac{(2k\sigma\cos\theta)^2}{(R_{10}+R_{20})^2}. \qquad (5.35)$$

Here α_0, β_0, R_{10}, R_{20} refer to the specular point R_0 in S, where $\frac{D-r_0}{R_{20}} - \frac{r_0}{R_{10}} = 0$, $r_0 = D\left(\frac{z_0}{z+z_0}\right)$. Also $\alpha_{z0} = \frac{z_0}{R_{10}} = \cos\theta$.

Observe that in (5.35) the term in the denominator which gives the spreading loss between source and receiver is no longer $R_1{}^2 R^2{}_2$ (compare (5.31)), but has switched to $(R_1 + R_2)^2$, which gives the smaller spreading loss observed in specular reflection from a plane.

Also note the occurrence of the Rayleigh parameter $2k\sigma\cos\theta$ (assumed small, recall) in the numerator of (5.35). Observe that here Lambert's law (scattering ~ $\cos^2\theta$) is applicable.

In the opposite limit, $k\ell << 1$, fine irregularities, (5.28) implies that

$$\langle |u(R)|^2 \rangle = \frac{C(k\ell_x)(k\ell_y)(k\sigma)^2}{\pi^2} \int_S \frac{\alpha_z{}^2\beta_z{}^2}{R_1{}^2R_2{}^2}\, dr \tag{5.36}$$

Clearly in this case, the scattering is very diffuse, with the incident angle θ of little importance, and the entire region S influencing the average intensity.

B) Rigid Surface[19]

Now suppose the initial condition is

$$\frac{\partial U}{\partial n}\left(r,\zeta(r)\right) = 0, \tag{5.37}$$

where the normal derivative $\partial/\partial n$ is given by

$$\frac{\partial}{\partial n} = \frac{\frac{\partial}{\partial z} - \gamma_x \frac{\partial}{\partial x} - \gamma_y \frac{\partial}{\partial y}}{\sqrt{1+\gamma_x{}^2+\gamma_y{}^2}} \tag{5.38}$$

Here $\gamma_x = \frac{\partial\zeta}{\partial x}$, $\gamma_y = \frac{\partial\zeta}{\partial y}$.

Again represent $U = \overline{U} + u$, the mean field plus a fluctuating field. Assuming $\zeta(r)$ is suitably small, expanding (5.37) in ζ, averaging and differencing, the boundary conditions (to first order) for $\overline{U}$, u become

$$\frac{\partial \overline{U}(r,0)}{\partial z} = \langle \gamma \cdot \nabla_r u \rangle - \zeta \langle \frac{\partial^2 u}{\partial z^2} \rangle \tag{5.39}$$

$$\frac{\partial u}{\partial z}(r,0) = \gamma \cdot \nabla_r \overline{U} - \zeta \frac{\partial^2 \overline{U}}{\partial z^2} \tag{5.40}$$

Here $\gamma = \nabla_r \zeta = (\gamma_x, \gamma_y)$, $\nabla_r = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$.

a) The Mean Field for a Rigid Surface

Using the Green's function G_+ in (4.4) for the mean plane z=0, the boundary condition (5.40) results in

$$u(R) = \frac{1}{2\pi} \int_{z'=0} \frac{e^{ik|R-r'|}}{|R-r'|} \{\zeta(r') \frac{\partial^2 \overline{U}(r')}{\partial z'^2} - \gamma(r') \nabla_{r'} \overline{U}(r')\} dr'. \tag{5.41}$$

This expression for u(R) can be used in conjunction with the boundary condition (5.39) to obtain an integral equation for $\overline{U}$ for z=0, analogous to (5.9) for the case of a free surface. It is not particularly revealing, however, and

will be omitted. Nevertheless, if one assumes an incident plane wave, with a reflected wave, as in (5.10), it can be shown that Snell's law continues to hold, that is, the reflected component of the mean field U for an incident plane wave propagates along the specular path.

If the surface is assumed isotropic, i.e., $W(\rho)$ depends only on $|\rho|$, the situation simplifies to the point where a formula for the reflection coefficient $V(\theta)$ can be obtained, namely, (compare with 5.11),

$$V(\theta) = \frac{\cos\theta - \eta_g(\theta)}{\cos\theta + \eta_g(\theta)}. \tag{5.42}$$

(θ is the specular angle between source and receiver.)

The effective impedance η_g is given by

$$\eta_g(\theta) = k^2\sigma^2 - \frac{i\sigma^2}{k}\int_0^\infty \frac{e^{ik}}{y}\{-k^3\sin\theta\cos\theta J_1(ky\sin\theta)W(y)$$
$$+ \frac{dW(y)}{dy}[(ik^3y-k^2\sin^2\theta)J_0(y) + \frac{k\sin\theta}{y}(1-iky-k^2y^2)J_1(ky\sin\theta)]\}dy. \tag{5.43}$$

As before, letting the correlation length ℓ be a scale factor, $W(y) = \tilde{W}(\frac{y}{\ell})$, various possiblities exist:

For $k\ell \ll 1$

$$\eta_g(\theta) \cong \frac{ik\sigma^2\sin^2\theta}{2\ell}\int_0^\infty \frac{1}{x}\tilde{W}(x)dx. \tag{5.44}$$

For $k\ell >> 1$, $\psi = \frac{\pi}{2} - \theta >> \frac{1}{\sqrt{k\ell}}$,

$$\eta_g(\theta) \cong k^2\sigma^2\cos^3\theta, \tag{5.45}$$

or, referring to (5.42)

$$V(\theta) = \frac{1-k^2\sigma^2\cos^2\theta}{1+k^2\sigma^2\cos^2\theta}. \tag{5.46}$$

Finally, for $k\ell >> 1$, and near grazing, $\psi\sqrt{k\ell} << 1$,

$$\eta_g(\theta) \cong \frac{\sigma^2\sqrt{k\ell}}{2\ell^2\sqrt{2\pi}} e^{\frac{i3\pi}{4}} \int_0^\infty \frac{1}{x^{3/2}} \frac{d\tilde{W}(x)dx}{dx} \tag{5.47}$$

A difference between the representation of the mean field for a free surface and a rigid surface occurs here because the reflection coefficient $V(\theta)$ for a rigid surface can have a pole near grazing, namely (see 5.42),

$$\cos\theta = -\eta_g(\theta) \tag{5.48}$$

Since $\left|\eta_g(\theta)\right| << 1$ in the cases considered here, a good approximation to a solution of (5.48) is given by setting $\eta_g = 0$, and iterating, with the result that the angle θ_p at which the pole occurs is given by

$$\cos\theta_p \cong -\eta_g\left(\frac{\pi}{2}\right) \tag{5.49}$$

The effect of the pole on $\overline{U}$ is as follows: For a point source above the boundary and at large distances, the reflected component of $\overline{U}$ has the form, for $\theta \neq \theta_p$,

$$\overline{U}_r(R) = V(\theta)\,\frac{e^{ik\overline{R}}}{\overline{R}} + 0\left(\frac{1}{k\overline{R}}\right). \tag{5.50}$$

where $\overline{R} = \sqrt{x_1{}^2 + y_1{}^2 + (z+z_0)^2}$ *, and θ is the specular angle between the source (at $(0,0,z_0)$) and receiver (at (x,y,z)), see Figure 5.

But if V has a pole (when $\theta \cong \theta_p$), then the reflected component has the form

$$\overline{U}_r(R) = \frac{e^{ik\overline{R}}}{\overline{R}}\left(1 + \eta_g\left(\tfrac{\pi}{2}\right)\sqrt{8k\overline{R}}\; e^{\frac{i\pi - s^2}{4}} \int_s^{i\infty} e^{t^2} dt\right), \tag{5.51}$$

where

$$s^2 = \frac{ik\overline{R}}{2} \cdot \eta_g{}^2\left(\tfrac{\pi}{2}\right)\left[1 + \frac{\psi}{\eta_g\left(\frac{\pi}{2}\right)}\right]^2 \tag{5.52}$$

The following asymptotic formulas are valid for grazing incidence, $\psi = 0$:

---

* Note that $\overline{R} = \left(1 + \frac{z}{z_0}\right)R_{10} = \left(1 + \frac{z_0}{z}\right)R_{20}$. See (5.35) and after.

$$\overline{U}_r(R) = \frac{e^{ik\overline{R}}}{\overline{R}} \cdot \begin{cases} 1+i\sqrt{\pi}s, & |s| << 1 \\ \frac{-1}{2s^2}, & |s| >> 1. \end{cases} \tag{5.53}$$

So if R, the position of the receiver, is such that the specular angle $\theta \cong \theta_p$, and if $|s|$ is large, the reflected mean field will experience additional attenuation.

However, $\theta_p < \frac{\pi}{2}$, and as grazing incidence approaches, $\theta_p < \theta \cong \frac{\pi}{2}$, the reflection coefficient approaches 1, the attenuation due to the pole disappears, and (5.50) is the appropriate estimate.

In fact, as will be shown in the next section, the average intensity for a rigid surface equals that for a free surface, at grazing incidence.

b) The Intensity of the Fluctuating Field for a Rigid Surface

The relevant formula for u(R) is (5.41), except now restrict the integral to the actual scattering surface S, so

$$u(R) = \frac{1}{2\pi} \int_S \frac{e^{ik|R-r'|}}{|R-r'|} \{\zeta(r') \frac{\partial^2 \overline{U}(r')}{\partial z'^2} - \gamma(r') \nabla_{r'} \overline{U}(r')\} dr'. \tag{5.54}$$

Introduce the same geometry as before, (see Figure 4), with R_1, R_2, α, β defined in (5.19), (5.20). As in the

case of a free surface, U may be replaced by the zero-order aproximation U_0, where

$$U_0(R) = \frac{e^{ik|R-S|}}{|R-S|} + \frac{e^{ik|R-S'|}}{|R-S'|} \tag{5.55}$$

as before (compare 5.21), where $S = (0,0,z_0) = -S'$, provided $|\cos\theta+n_g| > 0$, so that $V \cong -1$ is a valid approximation.

Then (5.54) can be rewritten in the form

$$u(R) = \frac{k^2}{\pi} \int_S \frac{(1-\alpha_\perp\cdot\beta_\perp)}{R_1R_2} e^{ik(R_1+R_2)}\zeta(r)dr. \tag{5.56}$$

But (5.56) immediately implies that

$$<|u(R)|^2> = \frac{k^4\sigma^2}{\pi^2} \int_S \frac{(1-\alpha_\perp\cdot\beta_\perp)^2}{R_1{}^2R_2{}^2} dr \int_{-\infty}^{+\infty} W(\rho)e^{-ik(\beta_\perp-\alpha_\perp)\cdot\rho}d\rho. \tag{5.57}$$

The assumptions made to derive (5.57) are the some as in the free surface case, (5.25) and (5.27).

But (5.56) has the same form as (5.26), with $(\alpha_z\beta_z)^2$ replaced by $(1-\alpha_\perp\cdot\beta_\perp)^2$, so estimates corresponding to (5.31) through (5.36) are valid in the case of a rigid surface also, with the above substitution.

Note that (5.35) is valid for rigid surfaces also, because at $r=r_0$, i.e., $\alpha_\perp=\beta_\perp$, $\alpha_{0z}\beta_{0z} = 1-\alpha_{0\perp}\cdot\beta_{0\perp}$, as shown

at the end of Appendix A. So as remarked earlier, if the scattering area is large and $k\ell \gg 1$, Lambert's law (5.35) is applicable to both a rigid and a free surface (for a small Rayleigh parameter, of course).

C) Surface with an Impedance Boundary Condition[19]

Assume the initial condition is

$$\frac{\partial U}{\partial n}(r,\zeta(r)) = -ik\eta U(r,\zeta(r)) \tag{5.58}$$

where n is the impedance at the interface between the upper and lower half spaces, and $\frac{\partial}{\partial n}$ is the normal derivative defined in (5.38).

Letting $U = \overline{U}+u$ as before, relatively few changes occur. The boundary condition for $\overline{U}$, 5.39, has a term $-ik\eta\overline{U}$ added on the right side. The effective impedance η_g is changed by the addition of the impedance η on the right side of (5.43).

And finally, the expression $\alpha_z{}^2\beta_z{}^2$ for a free surface, in (5.26), or $(1-\alpha_\perp\cdot\beta_\perp)^2$ in (5.57) is replaced by a more complicated expression $\left|\tilde{F}(\alpha,\beta,k,k_2,\ell,\ell_2\right|^2$ given in Appendix B. So the expressions in (5.31) through (5.36) are valid, with $\alpha_z{}^2\beta_z{}^2$ replaced by $\left|\tilde{F}\right|^2$. Note that in this case, (5.35) becomes

$$<\left|u(R)\right|^2 = \frac{4k^2\sigma^2\left|\tilde{F}(\alpha_0,\beta_0)\right|^2}{\beta_{0z}{}^2(R_{10}+R_{20})^2}. \tag{5.59}$$

VI. SCATTERING FROM SURFACES WITH LARGE-SCALE ROUGHNESS

The Kirchoff method or tangent plane solution[20].

To discuss scattering from a rough surface when there is no small parameter available, the following approximation is useful.

Assume the surface is so smooth that at each point r of the surface S the wave field U(r) may be represented as the sum of the incident field $U_0(r)$ and the field reflected from the tangent plane at r, that is,

$$U(r) = (1+V)U_0(r), \quad \frac{\partial U(r)}{\partial n} = (1-V)\frac{\partial U_0}{\partial n}(r) \tag{6.1}$$

Here $\frac{\partial}{\partial n}$ is the normal derivative, and V is the reflection coefficient.

If one assumes further that the incident wave is a plane wave at each point r of S, with a wave vector K depending on r, (6.1) becomes

$$U(r) = (1+V(\theta))U_0(r), \quad \frac{\partial U(r)}{\partial n} = i(K \cdot N)(1-V(\theta))U_0(r) \tag{6.2}$$

where N denotes the normal to S at r, θ is the angle between K and N, and the plane wave approximation is used for U_0, so that the normal derivative $\cong iK \cdot N\ U_0$.

The above representation requires that

$$\cos\theta \gg (ka)^{-1/3} \tag{6.3}$$

where a is the local radius of curvature of S at r. This condition results from the requirement that the tangent plane at r is a good approximation to S over a region whose dimensions are large relative to the wavelength of the incident wave.

Let the scattering surface S be complemented by a plane S' and the hemisphere C_R', (see Figure 6) so that a closed region is constructed containing all the field sources. Then if R is an arbitrary point within the region, by Green's theorem the field U(R) has the form

$$U(R) = U_0(R) + \frac{1}{4\pi}\int_{S+S'+C_{R'}} \Big\{U(r)\frac{\partial}{\partial n}\Big(\frac{e^{ik\tilde{R}_2}}{\tilde{R}_2}\Big) - \frac{e^{ik\tilde{R}_2}}{\tilde{R}_2}\frac{\partial}{\partial n}U(r)\Big\}dr. \tag{6.4}$$

But since all the sources are within the region, it follows that

$$\frac{1}{4\pi}\int_{S+S'+C_{R'}} \Big\{U_0(r)\frac{\partial}{\partial n}\Big(\frac{e^{ik\tilde{R}_2}}{\tilde{R}_2}\Big) - \frac{e^{ik\tilde{R}_2}}{\tilde{R}_2}\frac{\partial}{\partial n}U_0(r)\Big\}dr = 0. \tag{6.5}$$

Subtracting 6.5 from 6.4, the result is

$$U(R) = U_0(R) + \frac{1}{4\pi}\Big[\int_S + \int_{S'+C_{R'}}\Big]\Big\{(U-U_0)\frac{\partial}{\partial n}\Big(\frac{e^{ik\tilde{R}_2}}{\tilde{R}_2}\Big) - \frac{e^{ik\tilde{R}_2}}{\tilde{R}_2}\frac{\partial}{\partial n}(U-U_0)\Big\}dr. \tag{6.6}$$

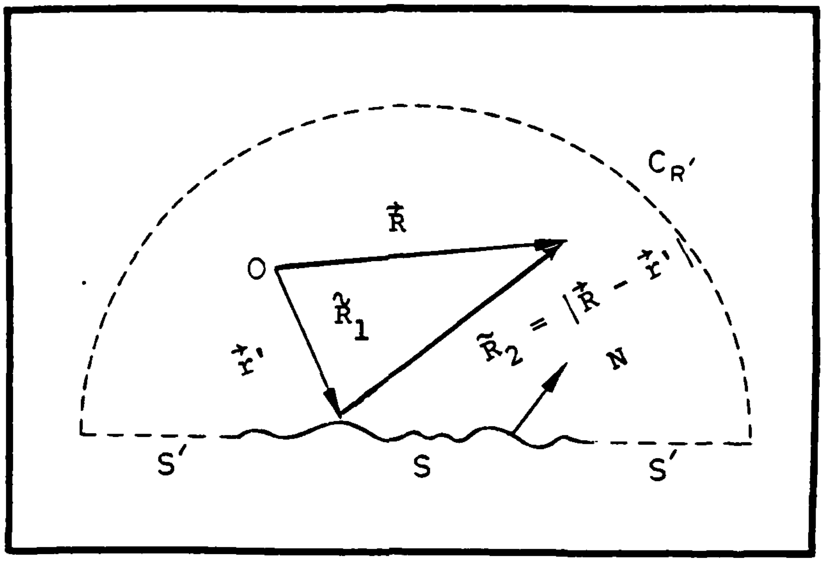

Figure 6

The Kirchoff approximation consists in assuming that on S', $U=U_0$, so the integral over S' is zero.

Further, assuming a radiation condition, that is, at infinity both U and $U_0 \sim \frac{e^{ikR'}}{R'}$, $R' \epsilon C_{R'}$, the integral over $C_{R'}$ may be neglected. Using 6.2 to represent $U-U_0$ on S, gives the result

$$U(R) = U_0(R) + \frac{1}{4\pi} \int_S V(r) \frac{\partial}{\partial n} \left(\frac{e^{ik\tilde{R}_2}}{\tilde{R}_2} U_0(r) \right) dr, \tag{6.7}$$

where V(r) is used to indicate the dependence of the reflection coefficient on the local angle of incidence. Here r is the length of $\vec{r}'$ along S, as seen in Fig. 7.

If the source is taken as a point source, then 6.7 may be written

$$U(R) = U_0(R) + \frac{1}{4\pi} \int_S V(r) \frac{\partial}{\partial n} \left[\frac{e^{ik(\tilde{R}_1+\tilde{R}_2)}}{\tilde{R}_1 \tilde{R}_2} \right] dr \tag{6.8}$$

where (see Figure 7), $\tilde{R}_1{}^2 = (z_0-\zeta(r))^2+|r|^2$, $\tilde{R}_2{}^2 = (z-\zeta(r))^2 + |D-r|^2$.

Now assuming $k\tilde{R}_1$, $k\tilde{R}_2 >> 1$, (the term used to describe this is that the source and receiver are in the wave zone relative to S), the integrand in (6.8) can be approximated by $-i(N \cdot \tilde{q}) \exp(ik(\tilde{R}_1+\tilde{R}_2))/(\tilde{R}_1\tilde{R}_2)$ where N is the normal to S at r, as shown for example in Fig. 6, and is given by

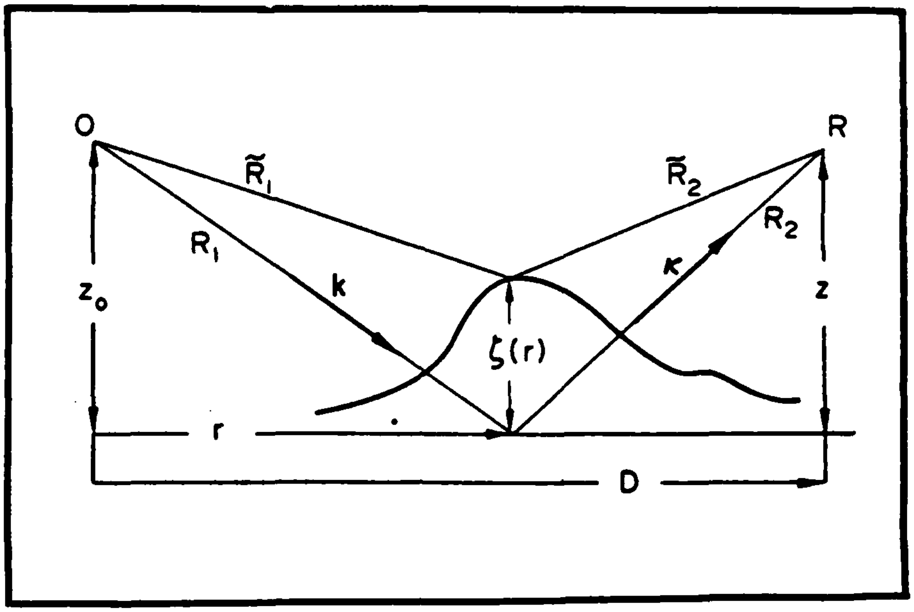

Figure 7

$$\gamma = \nabla_r \zeta(r), N_\perp = \frac{-\gamma}{\sqrt{1+|\gamma|^2}}, \; N_z = \frac{1}{\sqrt{1+|\gamma|^2}}, \text{ and } \tilde{q} = -k\nabla(\tilde{R}_1+\tilde{R}_2). \tag{6.9}$$

Referring to R_1, R_2 defined in (5.19), and α, β defined in (5.20), see Figure 4, if $k\sigma^2 \ll R_1, R_2$*, then

$$\tilde{R}_1 = R_1 + \alpha_z\zeta, \; \tilde{R}_2 = R_2 - \beta_z\zeta, \tag{6.10}$$

neglecting terms of order $\sigma^2/(R_1{}^2+R_2{}^2)$.

Under the same assumption

$$\tilde{q} = q = -k\nabla(R_1+R_2) = k(\beta-\alpha) \tag{6.11}$$

Using (6.10) and (6.11) in (6.8), and neglecting the incident field $U_0(R)$, then (6.8) becomes

$$U(R) = \frac{-i}{4\pi} \int_{S_0} \frac{V(r)(q_z - q_\perp\cdot\gamma)}{R_1R_2} \exp(i[k(R_1+R_2) - q_z\zeta(r)])dr, \tag{6.12}$$

where the integration has been taken over the projection S_0 of the surface S onto the plane z=0 $\left(dr_S = \frac{dxdy}{N_z}\right)$. Also, referring to 6.9, observe that $(N\cdot q)/N_z = q_z - q_\perp\cdot\gamma$. (As is true throughout this paper, $q_\perp$ is the projection of q the mean plane z=0.)

---

* More precisely $k\sigma^2|\alpha_z|^2 \ll R_1$, $k\sigma^2|\beta_z|^2 \ll R_2$. $|\alpha_z| = \cos\theta$, the local angle of incidence of the incident wave, and $|\beta_z| = \cos\theta'$, the local angle of incidence of the outgoing wave.

A) The Mean Field $\overline{U}$

a) "Small" Scattering Area

Suppose the dimensions L of the surface S are less than the dimensions of the Fresnel zone, that is

$$kL^2\cos^2\theta \ll R_1, \; kL^2\cos^2\theta' \ll R_2. \tag{6.13}$$

Then in (6.12), $k(R_1+R_2) \cong k(R_{10}+R_{20}) - q_\perp \cdot r$, where R_{10}, R_{20} are the distances between a fixed position on S and the source and receiver, respectively, and now q is a constant vector, over all of S. (6.12) then becomes

$$U(\alpha,\beta) = \frac{-ie^{ik(R_{10}+R_{20})}}{4\pi R_{10}R_{20}} \int_{S_0} V(r)(q_z-q_\perp\cdot\gamma)\exp\left(-i(q_\perp\cdot r+q_z\zeta(r)\right)dr. \tag{6.14}$$

Here $U(\alpha,\beta)$ rather than $U(R)$ is used to in indicate that (6.14) represents the field in the direction β after S is illuminated by an incident plane wave with wave vector $k\alpha$.

But as $\langle\gamma\rangle = 0$ and ζ and γ are independent, it follows directly that

$$\overline{U}(\alpha,\beta) = \frac{-iq_z Ve^{ik(R_{10}+R_{20})}}{4\pi R_{10}R_{20}} f_1(q_z) \int_{S_0} e^{-iq_\perp\cdot r}dr$$

where f_1 is the first order characteristic function defined in (1.6). But the integral above equals $(2\pi)^2\delta(q_\perp)$, and $q_\perp = 0$ only for the specular path connecting source and receiver, so finally

$$\overline{U}(\alpha,\beta) = \frac{-i\pi q_z V(\theta_0) e^{ik(R_{10}+R_{20})}}{R_{10}R_{20}} \delta(q_\perp) e^{-2k^2\sigma^2\cos^2\theta_0} \tag{6.15}$$

where θ_0 is the angle of specular reflection at $q_\perp = 0$. In (6.15), $f_1(q_z)$ was evaluated for a Gaussian surface, as $q_z = 2k\cos\theta_0$ when $q_\perp = 0$.

(6.15) again asserts that the mean field propagates along the specular path. Further, the effective reflection coefficient is $V(\theta_0)\exp(-2k^2\sigma^2\cos^2\theta_0)$, so that the coherent component of the scattered field decays exponentially with an increase in the Rayleigh parameter. Note that (5.16) is a special case of (6.15) for small Rayleigh parameter, for θ away from grazing, and $k\ell >> 1$.

b) "Large" Scattering Area

Now suppose S is so large that the inequalities in (6.13) are not satisfied. In this case, to discuss the mean field, return to equation (6.8), rather than continue from (6.12).

Assuming z_0, $z>\zeta$, the spherical waves in (6.8) may be expanded in plane waves, namely,

$$\frac{e^{ik\tilde{R}_1}}{\tilde{R}_1} = \frac{i}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{i(K\cdot r+[z_0-\zeta(r)]K_z)}}{K_z} dK \tag{6.16}$$

and

$$\frac{e^{ik\tilde{R}_2}}{\tilde{R}_2} = \frac{i}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{i(K\cdot(D-r)+[z-\zeta(r)]K_z)}}{K_z} dK \tag{6.17}$$

where $K = (K_x,K_y)$, and $K_z{}^2 = k^2 - |K|^2$.

As before, the normal derivative has the form given in 6.9.

Using 6.9, 6.16 and 6.17 in 6.8, and projecting the integration from S onto the mean plane $S_0(z=0)$, the result is

$$U(R) = U_0(R) + \frac{i}{16\pi^3} \int_{S_0} \Big[V(r) \iint_{-\infty}^{+\infty} \frac{[K_z+K'_z-\gamma\cdot(K-K')]}{K_z K_z'}$$

$$\exp\{i[(K-K')\cdot r + K'\cdot D + z_0 K_z + zK_z' - \zeta(r)(K_z+K_z')]\} dK dK'\Big] dr. \tag{6.18}$$

To calculate $\overline{U}$ from (6.18), again use that $\langle\gamma\rangle = 0$, and ζ and γ are independent random variables. For simplicity, set V=1. The average of $\exp(-i(K_z+K_z')(\zeta(r))) = f_1(K_z+K_z')$ as before, and the integral of $\exp(i(K-K')\cdot r)$ gives $(2\pi)^2\delta(K-K')$. The final result is

$$\overline{U}(R) = U_0(R) + \frac{i}{2\pi} \int_{-\infty}^{+\infty} \frac{f_1(2K_z)}{K_z} \exp[i(K\cdot D+(z_0+z)K_z] dK \tag{6.19}$$

Recalling that $f_1(2K_z) = \exp(-2k^2\sigma^2\cos^2\theta)$ defines the effective reflection coefficient in the previous section, (6.19) asserts that the mean field is the sum of the incident field $U_0(R)$ and the superposition of reflected plane waves, arriving from different angles, and with their own reflection coefficient.

Let $R' = \sqrt{|D|^2+(z_0+z)^2}$ be the distance from the mirror source $(0,0,-z_0)$ to the receiver at (D,z). Then it has been shown that (6.19) reduces to

$$\bar{U}(R) = U_0(R) + \frac{e^{ikR'}}{R'}\exp(-2k^2\sigma^2\cos^2\theta_0), \tag{6.20}$$

where θ_0 is the specular angle connecting source and receiver, if

$$2\left|(k\sigma)^4\sin^2\theta_0+(k\sigma)^2(3\cos\theta_0-1)\right| \ll kR'. \tag{6.21}$$

For $(k\sigma)^2$ small, (6.21) reduces to

$$\frac{k\sigma^2}{R'} \ll 1, \tag{6.22}$$

which is a very weak restriction on σ.

However, for $(k\sigma)^2 \gg 1$, and for θ_0 bounded away from 0 and $\pi/2$, that is,

$$\theta_0 \gg \frac{1}{k\sigma\sqrt{2}} \text{ and } \frac{\pi}{2} - \theta_0 \ll \frac{1}{2(k\sigma)}, \tag{6.23}$$

(6.21) is equivalent to

$$2(k\sigma)^4\sin^2 2\theta_0 \ll kR', \tag{6.24}$$

which is a good deal more stringent than (6.22).

This is due to the fact that for $(k\sigma)^2$ large, the effective reflection coefficient inside the integral in (6.19) will vary rapidly over S unless (6.24) is satisfied.

B) The Intensity of the Fluctuating Field

Returning to the representation of U(R) given in (6.12), note that for large Rayleigh parameter, here written $q_z\sigma$, the main contribution to the integral will come from the points of stationary phase, namely the points for which $\nabla_r[k(R_1+R_2)-q_z\zeta(r)] = 0$, that is

$$q_z\gamma + q_\perp = 0, \tag{6.25}$$

recalling that $\gamma = \nabla_r\zeta$, and $\alpha_\perp = \nabla_r R_1$, $\beta_\perp = -\nabla_r R_2$.

But the solution to (6.25), $\gamma_0 = -q_\perp/q_z$ defines a point of specular reflection between the source and receiver. So in (6.12), the term $V(r)(q_z-q_\perp\cdot\gamma)$ may be evaluated for γ a solution of (6.25). Since $q_z-q_\perp\cdot\gamma_0 = \left|q\right|^2/q_z$, (6.12) becomes

$$U(R) = \frac{-i}{4\pi}\int \frac{V(r)\left|q(r)\right|^2}{R_1R_2q_z}\exp(i[k(R_1+R_2)-q_z\zeta(r)])dr, \tag{6.26}$$

where V(r) now denotes the reflection coefficient evaluated at the angle of specular reflection at the point r.

But from (6.26), the following formula for the averaged intensity u results.

$$\begin{aligned}\langle\left|u(R)\right|^2\rangle &= \langle\left|U(R)\right|^2\rangle - \left|\langle U(R)\rangle\right|^2 \\ &= \frac{1}{(4\pi)^2}\int_{S_0}\int_{S_0}\frac{VV^{*\prime}\left|q\right|^2\left|q'\right|^2}{R_1R_1'R_2R_2'q_zq_z'}\exp[ik(R_1+R_2-R_1'-R_2']\cdot \qquad (6.27)\\ &\quad [\langle\exp(-iq_z\zeta(r)+iq_z'\zeta(r'))\rangle - \langle e^{-iq_z\zeta(r)}\rangle\langle e^{iq_z'\zeta(r')}\rangle]drdr'.\end{aligned}$$

All variables with primes refer to r'.

Let $\rho = r'-r$. Then, as before, see (5.24),

$$R_1+R_2-R_1'-R_2' \cong \left[\frac{D-r}{R_2} - \frac{r}{R_1}\right]\cdot\rho = q_\perp\cdot\rho, \tag{6.28}$$

where quadratic terms in ρ are neglected, which is valid under conditions to be specified subsequently.

Then, referring to (1.6) and (1.7), (6.27) becomes

$$<|u(R)|^2> = \frac{1}{(4\pi)^2}\int_{S_0}\frac{|V(r)|^2|q(r)|^4}{R_1{}^2R_2{}^2q_z{}^2}\int_{-\infty}^{+\infty} e^{iq_\perp\cdot\rho}[f_2(q_z,-q_z;\rho)$$

$$- f_1{}^2(q_z)]d\rho dr. \tag{6.29}$$

In (6.29), the terms before the exponential in (6.27) were evaluated at $\rho=0$, or $r'=r$. Conditions for the validity of this will also be given below. Observe also that in (6.29), the ρ-limits of integration are extended over the infinite plane.

If S is Gaussian, the expression in the bracket in (6.29) becomes (refer to (1.9)),

$$f_2(q_z,-q_z;\rho)-f_1{}^2(q_z) = \exp[-q_z{}^2\sigma^2(1-W(\rho))]-\exp(-q_z{}^2\sigma^2) \tag{6.30}$$

The validity of all of the transformations and approximations in going from (6.27) to (6.29) depend upon the dimensions of the ρ-region over which the expression in (6.30) diminishes, so that portions of the ρ-plane outside this region may be neglected.

For large Rayleigh parameter, $(q_z\sigma)^2 \gg 1$, expanding $W(\rho)$ in a Taylor series, the result is that (6.29) is valid for

$$k R \Gamma_x{}^2 \gg 1, \; k R \cos^2\theta \Gamma_y{}^2 \gg 1 \tag{6.31}$$

where $R \cong R_1, R_2, \cos^2\theta \cong \cos^2\theta'$.*

Interpreting the inequalities in (6.31) geometrically, the result is that for $(q_z\sigma)^2 \gg 1$, (6.29) is valid if the region where specular reflections dominate substantially exceeds in size the first Fresnel zone.

If now we consider the case $(q_z\sigma)^2 \ll 1$, the expression in (6.30) reduces to $q_z{}^2\sigma^2 W(\rho)$, and in this case (6.29) nearly coincides with (5.26). A factor $|q|^4/16$ occurs in (6.29), and a term $k^4\alpha_z{}^2\beta_z{}^2$ in (5.26). These agree when $\alpha_\perp = \beta_\perp$, $\alpha_z = -\beta_z$ (scattering in the specular direction), but not elsewhere. Further the conditions for validity of (6.29) turn out to be (5.25) and (5.27), that is, the region where specular reflections dominate must be much smaller than the Fresnel zone.

Not only is (6.29) valid at reasonable ranges for both large and small values of the Rayleigh parameter, but as will be shown below, (6.29) reduces to the correct limiting value as the mean slope of S approaches zero. Further, for large values of the Rayleigh parameter, a limiting form of (6.29) can be derived involving the slope probability density function which agrees with results derived using an alternative approach to scattering.

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* It can be shown that (6.29) holds if

$$(kR)^2\Gamma_x{}^3 \gg 1, \; (kR)^2(\cos\theta\Gamma_y)^3 \gg 1,$$

which is weaker than (6.31).

For all of these reasons, (6.29) is a fundamental result of scattering theory.

a) Small Scattering Surface

Now suppose that the dimensions of the scattering surface S, and its projection S_0 are sufficiently small that the r-dependence in the integral in (6.29) may be neglected. Then (6.29) becomes

$$J(\alpha,\beta) = \frac{|V|^2 |S_0| |q|^4}{(4\pi R_1 R_2 q_z)^2} \int_{-\infty}^{+\infty} e^{-iq_\perp \cdot \rho} [f_2(q_z,-q_z;\rho) - f_1{}^2(q_z)] d\rho \tag{6.32}$$

Here as in Section V, the notation $J(\alpha,\beta)$ is used as a reminder that the average intensity is now a function of the incoming and outgoing wave vectors, α (in), β (out), and $q = k(\beta-\alpha)$. $|S_0|$ is the area of S_0.

For $(q_z\sigma)^2 \gg 1$, (6.32) is valid if

$$\frac{L_x \cos\theta}{R_1}, \frac{L_x \cos\theta'}{R_2} \ll \Gamma_x; \; R_1\cos\theta, \; R_2\cos\theta' \ll \frac{L_y}{\Gamma_y} \tag{6.33}$$

where L_x, L_y are x-y dimensions of S_0.

For $(q_z\sigma)^2 \ll 1$, (6.32) is valid if

$$\frac{kL_x \ell_x \cos^2\theta}{R_1} \ll 1, \quad \frac{kL_y \ell_y}{R_1} \ll 1, \tag{6.34}$$

plus similar inequalities with R_1,$\cos\theta$ replaced by R_2,$\cos\theta'$. (6.34) corresponds to the restriction (5.30), derived earlier for the perturbation solution.

Now consider (6.32) for $(q_z\sigma)^2 >> 1$. Then the term $f_1{}^2(q_z) = \exp(-q_z{}^2\sigma^2)$ can be neglected. Further, as soon as $1-W(\rho)$ becomes significantly different from zero, the f_2 term (see (6.30)) is also essentially zero. So for large $q_z\sigma$, only the region near $\rho=0$ needs to be considered.

But then, taking a Taylor expansion of $1-W(\rho)$ around $\rho=0$, and evaluating the integral for large $(q_z\sigma)$, using the method of stationary phase, (or actually, Laplace's method here), the result is

$$J(\alpha,\beta) = \frac{|V|^2|S_0||q|^4}{(4\pi R_1 R_2 q_z)^2} \cdot \frac{2\pi}{q_z{}^2\sigma^2\sqrt{|W_{xx}(0)W_{yy}(0)|}}$$

$$\cdot \exp\left[\frac{1}{2q_z{}^2\sigma^2}\left(\frac{q_x{}^2}{W_{xx}(0)} + \frac{q_y{}^2}{W_{yy}(0)}\right)\right] \qquad (6.35)$$

But setting $W_{xx}(0) = \frac{-1}{\ell_x{}^2}$, $W_{yy}(0) = \frac{-1}{\ell_y{}^2}$, and $\Gamma_x{}^2 = \sigma^2/\ell_x{}^2$, $\Gamma_y{}^2 = \sigma^2/\ell_y{}^2$, (6.35) can be rewritten as

$$J(\alpha,\beta) = \frac{|V|^2|S_0||q|^4}{4R_1{}^2R_2{}^2q_z{}^4} w_2(\gamma_0), \qquad (6.36)$$

where $w_2(\gamma)$ is the slope probability density function defined in (1.2), and $\gamma_0 = -q_\perp/q_z$ is the facet slope providing specular reflection between source and receiver.

The derivation of (6.36) requires that

$$(q_z\sigma)^2 > 1 \text{ if } \frac{|q_\perp|}{q_z\Gamma} < 1$$

and (6.37)

$$(q_z\sigma)^2 >> \left(\frac{|q_\perp|}{q_z\Gamma}\right)^4 \text{ if } \frac{|q_\perp|}{q_z\Gamma} > 1.$$

Here $\Gamma = \max(\Gamma_x, \Gamma_y)$.

Note that the expression in (6.36) is independent of k. This is due to the fact that (6.36) expresses the high frequency limit of the integral in (6.32).

Radar backscattering studies from the moon exhibit some frequency dependence, even near the specular direction[21], so to that extent (6.36) is not correct.

Further, the derivation of (6.36) given requires very little information regarding $W(\rho)$, only that second derivatives exist. (But see an alternate derivation of (6.36) below.) So if the scattering depends upon more than three parameters σ, ℓ_x, ℓ_y, then (6.36) is too simple.

However, its very simplicity is appealing. Further, the fact that the slope density function w_2 is evaluated at $\gamma_0 = -q_\perp/q_z$ is also interesting. A facet with slope γ_0 is such that the wave vectors α (from the source), and β (to the receiver) make equal angles with the normal to the facet. So the reflection from such a plane is specular reflection from source to receiver. Thus (6.36) can be interpreted as the product of a geometric term $|q|^4/q_z^4$, an area term, a reflection coefficient and a spreading loss term, all multiplied by the probability that there is a facet

suitably oriented to provide specular reflection between source and receiver.

This is an expression of the idea that for a large Rayleigh parameter, the scattering is due to glints from properly oriented facets of the surface. And since the mean plane is flat, $\langle\zeta\rangle = 0$, this probability is highest near the mean specular path, so this term should dominate there.

(6.36) is of sufficient interest that an alternative derivation will now be given.

Begin with $U(\alpha,\beta)$ as given in (6.14).

Then

$$\left|U(\alpha,\beta)\right|^2 = \frac{1}{(4\pi R_1 R_2)^2} \int_{S_0} \int_{S_0} \left|V\right|^2 (q_z - q_\perp \cdot \gamma)(q_z - q_\perp \cdot \gamma') \cdot \exp\left[-iq_\perp(r-r') + q_z(\zeta(r) - \zeta(r'))\right] dr\,dr' \tag{6.38}$$

Now in the exponent, let $\zeta(r) - \zeta(r') \cong \nabla_r \zeta(r-r') = -\gamma \cdot \rho$, where $\rho = r' - r$ as before. Neglecting the ρ dependence in γ' results in a ρ integral of the form

$$\int e^{-i(q_\perp + q_z\gamma)\cdot\rho} d\rho = (2\pi)^2 \delta(q_\perp + q_z\gamma), \tag{6.39}$$

again assuming that the integral is dominated by the contribution near $\rho=0$, so the integral may be extended to the infinite plane.

Introducing (6.39) into (6.38), and then averaging with respect to the random variable γ, the result is

$$J(\alpha,\beta) = \langle |U(\alpha,\beta)|^2\rangle = \frac{|S_0|}{4R_1{}^2R_2{}^2}\int |V|^2(q_z - q_\perp \cdot \gamma)^2 \delta(q_\perp + q_z\gamma) w_2(\gamma)d\gamma$$

$$= \frac{|V(\theta)|^2 |S_0| |q|^4}{4R_1{}^2R_2{}^2 q_z{}^4} w_2(\gamma_0), \tag{6.40}$$

where $\gamma_0 = -q_\perp/q_z$ as before. θ_0 is the specular angle relative to the normal to the facet with slope γ_0, that is, $\cos\theta_0 = \frac{N \cdot q}{|q|}$. In (6.39), use was made of the fact that $(q_z - q_\perp \cdot \gamma_0)^2 = |q|^4/q_z{}^2$. The estimates required in deriving (6.40) are precisely the inequalities (6.37) obtained in deriving (6.36).

Note that in deriving (6.40), no mention was made of the correlation function W. However, the underlying premise throughout the whole section is that the surface S is smooth enough that the tangent plane approximation makes sense. But as remarked earlier, a reasonable smooth surface requires W to have second derivatives. So the derivation leading to (6.40) is not more general than that for (6.36).

b) Shadowing for Small Scattering Surface[22]

In deriving the above relations, such as the formula for $|U(\alpha,\beta)|^2$ in (6.38), the implicit assumption was made that the entire surface S was illuminated, so that the integration could be taken over the entire projected area S_0.

However, for rough surfaces, with steep slopes, and near grazing, obviously some portions of the surface are in shadow, and do not interact with the incident wave. To

correct this deficiency, a factor relating to the probability of being shadowed should be introduced.

The simplest way to introduce the shadowing correction is to assume no diffraction and modify (6.38) so that the integration is now over $S_{ill} x S_{ill}$, rather than $S_0 x S_0$, where S_{ill} is the projection of the illuminated portion of S. Continuing as above, approximating $\zeta(r)-\zeta(r') \cong -\gamma \cdot \rho$, and using (6.39), we arrive at (6.40). But now, rather than computing the average with respect to γ using the density function $w_2(\gamma)$, the γ average is computed using the effective slope density function $w_e(\gamma;\alpha,\beta)$ which is the probability of the occurrence of a facet with slope γ for which an incoming ray α is not intercepted by some other portion of the surface, and for which the outgoing ray β is likewise unshadowed.

Now the result is

$$w_e(\gamma;\alpha,\beta) = w_2(\gamma) P(\gamma;\alpha,\beta) \tag{6.41}$$

where $P(\gamma;a,\beta)$ is the probability that neither of the rays α or β reaching the facet with slope γ intersect the surface elsewhere.

The only case where theory has produced an evaluation of $P(\gamma;\alpha,\beta)$ which has been verified experimentally is when α, β lie in the same vertical plane, the plane of incidence[23].

Let $\alpha = (\sin\theta\cos\phi, \sin\theta\sin\phi, -\cos\theta)$, and $\beta = (\sin\theta'\cos\phi', \sin\theta'\sin\phi', \cos\theta')$, where θ, θ' are the incidence angle of α, β with respect to the normal to the mean plane, and ϕ, ϕ' are the angles α, β make with respect to the x-axis, (see Figure 3).

If $\phi' = \pi+\phi$ (backscattering) and $\bar{\theta} = \max(\theta,\theta')$,

$$P(\gamma;\alpha,\beta) = H(\cot\bar{\theta}-\gamma_x\cos\phi-\gamma_y\sin\phi)\cdot\frac{1}{1+\Lambda(\bar{\theta},\phi)}. \qquad (6.41)$$

If $\phi' = \phi$ (forward scattering)

$$P(\gamma;\alpha,\beta) = H(\cot\theta'-\gamma_x\cos\phi-\gamma_y\sin\phi')H(\cot\theta+\gamma_x\cos\phi+\gamma_y\sin\phi)$$

$$\cdot\frac{1}{1+\Lambda(\theta,\phi)+\Lambda(\theta',\phi')}. \qquad (6.42)$$

Here $H(x)$ is the unit step function, 1 for $x>0$, 0 for $x<0$. $\Lambda(\theta,\phi)$, for a Gaussian surface, is defined by

$$\Lambda(\theta,\phi) = \frac{1}{2a}\left[\sqrt{\frac{2}{\pi}}\, e^{\frac{-a^2}{2}}-a\mathrm{Erfc}\left(\frac{a}{\sqrt{2}}\right)\right]. \qquad (6.43)$$

where $a^2 = \cot^2\theta/(\Gamma_x{}^2\cos^2\phi+\Gamma_y{}^2\sin^2\phi)$.

The difference between the two cases is as follows. In (6.41), (backscattering), if the lower ray is not shadowed, then certainly neither is the upper ray, that is, the two events $\{\alpha$ not shadowed$\}$ and $\{\beta$ not shadowed$\}$ are related, one is a subset of the other.

However, in forward scattering, (6.42), the assumption is made that the two events are independent, which seems reasonable in this case.

No shadowing theory has been developed for an arbitrary relation between ϕ and ϕ'. The difficulty lies in making the transition between two independent shadowing events, and the totally dependent arrangement in backscattering.

In the absence of any analysis of the transition, the suggestion has been made to use (6.42), treating the events as independent, as long as β lies outside some wedge containing α, i.e., as long as $|\phi'-\phi| > \varepsilon_0 > 0$[24]. Presumably ε_0 need not be very large, e.g., $\pi/12$; although Bass suggests $\varepsilon_0 = \frac{\pi}{2}$. For the reverse inequality, (6.41) should be used.

One further note, in the final form of (6.40), where $w_2(\gamma_0)$ is replaced by $w_e(\gamma_0;\alpha,\beta)$, the step functions in (6.41) and (6.42) are one, as the arguments are positive.

But when these results are extended to a large scattering surface, these step functions will play a role.

The shadowing theory presented thus far is based upon the approximation in (6.38) of $\zeta(r)-\zeta(r')$ by $-\gamma\cdot\rho$. This eliminates the random variable ζ in favor of γ. And the introduction of a correction for shadowing is reasonably well understood, when averaging over the slope vector γ.

However, as remarked earlier, the formula (6.32), [or (6.29)], seems more fundamental than (6.40), in that (6.32) will give either (6.40) or the perturbation result (5.31), according to the size of $q_z\sigma$. But to the author's knowledge, no shadowing correction has been applied to (6.32).

There is a formula for $w_e(\zeta;\alpha,\beta)$ analogous to (6.41), where $w_e(\zeta;\alpha,\beta)$ is the probability density function of the surface height, given that the rays α,β are not shadowed, namely.

$$w_e(\zeta;\alpha,\beta) = w(\zeta)P(\zeta;\alpha,\beta) \tag{6.44}$$

where

$$P(\zeta;\alpha,\beta) = \left(1+\Lambda(\theta,\phi)+\Lambda(\theta',\phi')\right)\cdot\left[\int_{-\infty}^{\zeta} w(\zeta')d\zeta'\right]^{[\Lambda(\theta,\phi)+\Lambda(\theta',\phi')]}$$

$$\cdot \int\int_{-\infty}^{+\infty} P(\gamma;\alpha,\beta)d\gamma \tag{6.45}$$

The fact that $P(\zeta;\alpha,\beta)$ depends upon ζ will prevent the averaging of the term $\zeta(r)-\zeta(r')$ giving f_2, as before in (6.32).

One suggestion for introducing a shadowing correction into (6.32) or (6.29) is to approximate an integral over S_{ill} as follows:

$$\left\langle \int_{S_{ill}} \{\ \} dr \right\rangle = \int_{S_0} P(\alpha,\beta) \langle\{\ \}\rangle dr,$$

where $P(\alpha,\beta)$ is the probability of the rays α, β reaching an illuminated portion of the surface independently of the height ζ, and slope γ.

For an isotropic Gaussian surface, and for α,β independently shadowed,

$$P(\alpha,\beta) = \frac{\int_{-\cot\theta}^{\cot\theta'} w_2(\gamma)d\gamma}{1+\Lambda(\theta)+\Lambda(\theta')}. \tag{6.46}$$

If the shadowing of α and the shadowing of β are dependent, then

$$P(\alpha,\beta) = \frac{\int_{-\infty}^{\cot\bar{\theta}} w_2(\gamma)d\gamma}{1+\Lambda(\bar{\theta})}, \tag{6.47}$$

where $\bar{\theta} = \max(\theta,\theta')$ as before.

The shadow corrected version of (6.32) is then

$$J(\alpha,\beta) = J_0(\alpha,\beta)P(\alpha,\beta), \tag{6.48}$$

where $J_0(\alpha,\beta)$ is given in (6.32).

For a non-isotropic surface, the integrals in the numerator in (6.46) and (6.47) are replaced by the integral of $w_2(\gamma_x,\gamma_y)$ restricted by the step functions appearing in (6.42) and (6.41), respectively.

b) Large Scattering Surface

If the inequalities (6.34) are not valid, so that a large region is involved in the scattering, then the r integral needs to be evaluated. The result is

$$<|u(R)|^2> = \int_{S_0} \frac{J(\alpha,\beta)}{|S_0|}dr \tag{6.49}$$

where $J(\alpha,\beta)$ is given in (6.32), or for large $(q_z\sigma)$, in (6.40), where no shadowing correction is made; or by (6.40) multiplied by $P(\gamma;\alpha,\beta)$, or (6.48), which is (6.32) multiplied by $P(\alpha,\beta)$.

It is interesting to consider the limit of $<|u(R)|^2>$ as $\Gamma^2 \to 0$ (but letting $kR\to\infty$ so (6.31) is still satisfied). Let us write (6.49) in the form

$$\langle |u(R)|^2\rangle = \frac{1}{(4\pi)^2}\int_{S_0} \frac{P(\alpha,\beta)|V(r)|^2|q|^4}{(R_1R_2q_z)^2}\int_{-\infty}^{+\infty} e^{-iq_\perp\cdot\rho}[f_2(q_z,-q_z;\rho)$$
$$- f_1{}^2(q_z)]d\rho dr \qquad (6.50)$$

As $\Gamma^2\to 0$, f_2 (see (6.30)) becomes sharply peaked at $\rho=0$, so

$$\int_{-\infty}^{+\infty} e^{-iq_\perp\cdot\rho}[f_2(q_z,-q_z;\rho) - f_1{}^2(q_z)]d\rho \cong (2\pi)^2\delta(q_\perp)[1-f_1{}^2(q_z)] \qquad (6.51)$$

But as in (5.35) $q_\perp=0$ occurs at the point of specular reflection with respect to the mean plane, namely at $r_0 = D\cdot(\frac{z_0}{z_0+z})$.

The result is

$$\langle |u(R)|^2 = \frac{\frac{1}{4}|V(\theta_0)|^2(1-f_1{}^2(q_z))}{(R_{10}+R_{20})^2}, \qquad (6.52)$$

where we have used the fact that at $q_\perp=0$, $|q|=q_z=\beta_z$. θ_0, as before, is the specular angle at r_0, $\cos\theta_0 = \frac{z_0}{R_{10}} = \frac{z}{R_{20}}$.

When $(q_z\sigma)^2 >> 1$, $f_1{}^2\cong 0$, and (6.52) gives the correct limiting result in specular reflection from a plane.

In (6.52), the probability $P(\alpha,\beta)\to 1$ as $\Gamma^2\to\infty$, as $\Lambda(\theta)\to 0$ as $\Gamma^2\to\infty$.

VII. EXTENSIONS TO THE THEORY: COMPOSITE SURFACES[25]

There are basically two solutions to the rough surface scattering problem that have gained acceptance. One is the perturbation solution (5.31), for a small Rayleigh parameter, and the other is (6.40), for a large Rayleigh parameter, using the tangent plane approximation. (Actually (6.40) should be replaced by the shadowed version, with $w_2(\gamma)$ replaced by $w_e(\gamma;\alpha,\beta)$.)

As has been remarked several times already, both (5.31) and (6.40) can be obtained from (6.32). (At least the free surface perturbation solution can be obtained from (6.32), to within a geometric multiplier, although since the rigid surface solution coincides with the free surface solution at grazing incidence, perhaps the difference is slight.)

However, (6.32) has not been used extensively as a scattering solution, probably because a Fourier transform must be evaluated, whereas (5.31) and (6.40) are simple function evaluations.

Now (6.40 - 6.44), and (5.31) are valid in different, nearly complementary regions of the parameter space (k,α,β,σ), namely the geometrical optics solution is valid for large $k\sigma$, near the specular angle, while the perturbation solution is valid for small $k\sigma$, and/or near grazing, away from the specular angle.

Clearly it would be desirable to have a simple solution which would bridge the gap between the two existing

solutions, as the result would be a theory valid over all parameter space.

Note that the two solutions are responding to different features of the scattering surface. The geometrical optics solution is describing specular reflection from properly oriented planar facets of the surface, while the perturbation solution expresses the diffraction from smaller features of the surface.

This notion led to the introduction of a composite surface, that is, expressing the surface as a sum of a large and a small surface,

$$z = \zeta(r) = \zeta_L(r) + \zeta_s(r) \tag{7.1}$$

where in (7.1) and subsequently, the subscripts L and s refer to the large and small surfaces respectively.

The idea here is the large surface contains the planar facets, (long waves in surface), and superimposed on these are the small ripples (short waves) responsible for diffraction.

Again note the utility of assuming a Gaussian surface, for if both ζ_L and ζ_s are Gaussian, so is ζ.

The first person to use this notion of composite surface was Kuryanov[26]. He suggested modifying the small surface (perturbation) solution in two ways: i) Express the geometric factor in the solution relative to the normal to the large surface ($N_L = N(\gamma_L)$), rather than in terms of

the vertical normal to the mean plane; and ii) since the orientation of the large facets is random, average the modified perturbation solution in (i) with respect to γ.

Kuryanov wrote before an adequate shadowing theory was developed, which limited his results. However, Brown[2] recently independently rederived Kuryanov's results, including shadowing.

An interesting way to develop Brown's results is due to Dashen[3]. Begin with (6.32), repeated here for reference.

$$J(\alpha,\beta) = \frac{|V|^2 |S_0| |q|^4}{(4\pi R_1 R_2 q_z)^2} \int_{-\infty}^{+\infty} e^{-iq_\perp \cdot \rho}[\exp(-q_z \sigma^2(1-W(\rho))-e^{-q_z{}^2\sigma^2}]d\rho. \tag{7.2}$$

Now (7.1) implies

$$\sigma^2 W(\rho) = \sigma_L{}^2 W_L(\rho)+\sigma_s{}^2 W_s(\rho). \tag{7.3}$$

Assuming $(q_z\sigma_s)^2$ is small, then

$$\exp(q_z{}^2\sigma_s{}^2 W_s(\rho)) \cong 1+q_z{}^2\sigma_s{}^2 W_s(\rho). \tag{7.4}$$

Then, neglecting the term $e^{-q_z{}^2\sigma^2}$ in (7.2), we can write

$$J(\alpha,\beta) = \frac{|V|^2 |S_0| |q|^4}{(4\pi R_1 R_2 q_z)^2} \Big[\int_{-\infty}^{+\infty} e^{-iq_\perp \cdot \rho} e^{-q_z{}^2\sigma_L{}^2(1-W_L(\rho))} d\rho \tag{7.5}$$

$$+ q_z{}^2 \int_{-\infty}^{+\infty} e^{-iq_\perp \cdot \rho}(e^{-q_z{}^2\sigma_L{}^2(1-W_L(\rho))})\cdot\sigma_s{}^2 W_s(\rho)d\rho\Big]$$

But the second term in (7.3), the Fourier transform of the product, becomes the convolution of the transform of the two factors.

Now suppose that $(q_z\sigma)^2$ is large enough so that, as in (6.35),

$$\int_{-\infty}^{+\infty} e^{-ir\cdot\rho}\cdot e^{-q_z{}^2\sigma_L{}^2(1-W_L(\rho))}d\rho = \frac{4\pi^2}{q_z{}^2}\, w_{eL}(r/q_z;\alpha,\beta) \tag{7.4}$$

where a shadowing correction has been introduced by using w_{eL}, defined in (6.41), rather than w_{2L}.

Using (7.4) in the convolution form of (7.3), the result is

$$\begin{aligned} J(\alpha,\beta) = {} & \frac{|V|^2|S_0||q|^4}{R_1{}^2R_2{}^2q_z{}^4} w_{eL}(\gamma_0;\alpha,\beta) \\ & + \frac{|V|^2|S_0||q|^4}{4R_1{}^2R_2{}^2}\int_{-\infty}^{+\infty} F_{1s}(q_z(\gamma-\gamma_0))w_{eL}(\gamma;\alpha,\beta)d\gamma, \end{aligned} \tag{7.5}$$

recalling that $\gamma_0 = -q_\perp/q_z$, and in the last integral the dummy variable in the convolution has been set equal to $q_z\gamma$.

On comparing the last term in (7.5) with (5.31), it is clear that this term is a shadow corrected averaged version of (5.31). In fact, if $k\ell >> 1$, then, (just as in

the transition from (5.31) to (5.33)), $w_{eL}(\gamma;\alpha,\beta) \cong \delta(0)$, (as $\Gamma_L{}^2 \to 0$ also), and $F_{1S}(-q_z\gamma_0) \cong \delta(q_\perp)$. Thus this term in (7.5) reduces to (5.33) also.

Clearly this last term in (7.5) incorporates Kuryanov's notion of averaging the small surface result with respect to the slopes of the large surface. However, the geometric term $|q|^4$ is not included in the averaging.

Using a different approach, Brown included the geometric term in the convolution. If (7.5) was modified by taking the $|q|^4$ term within the γ-integral, written in the form

$$|q|^4 = (|q_\perp|^2 + q_z{}^2)^2 = q_z{}^4(|\gamma|^2+1)^2, \tag{7.6}$$

this gives one interpretation of expressing the geometric factor in terms of the slope γ of the facet of the large surface.

A more fundamental version of (7.5) could be provided by replacing the term $|q|^4$ by the correct expression for the perturbation solution, i.e., $\alpha_z{}^2\beta_z{}^2$ for a free surface, $(1-\alpha_\perp\cdot\beta_\perp)^2$ for a rigid surface, or more generally $|\tilde{F}(\alpha,\beta)|^2$. However, in line with Kuryanov's notion of expressing these factors in terms of the normal to the facet with slope γ, let α_z, β_z be interpreted as the component along the normal $N(\gamma)$, and $\alpha_\perp$, $\beta_\perp$ the perpendicular component.

That is,

$$\alpha_z = (\alpha\cdot N), \text{ and } \alpha_\perp = \alpha-(\alpha\cdot N)N. \tag{7.7}$$

Recall that $N(\gamma) = (1,-\gamma)/\sqrt{|\gamma|^2+1}$.

As remarked earlier after (6.31), $|q|^4$ should be replaced by $16k^4\alpha_z{}^2\beta_z{}^2$. Accordingly, the second term in (7.5) has the form

$$\frac{4k^4 |V|^2 |S_0|}{R_1{}^2 R_2{}^2} \int_{-\infty}^{+\infty} \alpha_z(\gamma)^2 \beta_z(\gamma)^2 F_{1s}(q_z(\gamma-\gamma_\partial)) w_{2L}(\gamma)\, d\gamma. \qquad (7.8)$$

This is essentially the form used by Andreeva et al[29], and before that Bachmann[30], and Kuryanov[26]. Observe that in the integral w_{eL} has been replaced by w_{2L}, as earlier Andreeva et al[31] showed that at low grazing angles the shadowing correction is not important, except possibly at very high frequencies. Since the modification and averaging of $|q|^4$ over γ is only a significant correction at low grazing angles, it is consistent to replace w_{eL} by w_{2L} above. Neither Bachmann or Kuryanov had a significant amount of data with which to test their model. In reference 29, however, scattering data for grazing angles from 3° to 10° was available, at several frequencies and wind speeds, and the corrections introduced in (7.8) (for the case of back-scattering) produced a model which fit the data rather well.

Probably the various versions of the composite surface solution, ((7.5), or the substitution suggested in (7.6), or (7.7)), are all essentially equivalent, as $w_{eL}(\gamma;,\alpha,\beta)$ for moderate values of $\Gamma_L{}^2$ will be concentrated near $\gamma=0$, where the above expressions differ very little.

One must observe, as does Brown, that the integral term in (7.5) should include the step functions (see (6.41) or (6.42)) in the definition of w_{eL}, in order to be rigorous. Other than complicating the evaluation of the convolution, it is not clear whether the correction so introduced is signficant. Some numerical experience would be useful here.

A more interesting question relates to (7.1), the division of the surface into large and small.

The basic assumption here of course is that $(k\sigma)^2 >> 1$, for otherwise the perturbation solution would probably be valid.

Given this, the division of large and small is given by dividing the spectrum at a value k* to be chosen.

$$F_{1_L}(K) = F_1(K)H(k_* - |K|) \tag{7.9}$$

and

$$F_{1_S}(K) = F_1(K)H(|K| - k_*) \tag{7.10}$$

Recall H is the unit step function, one for positive argument, zero for negative.

Then

$$\sigma_L{}^2 = \int_{-\infty}^{+\infty} F_{1_L}(K)dK \tag{7.11}$$

$$\sigma_S{}^2 = \int_{-\infty}^{+\infty} F_{1_S}(K)dK \tag{7.12}$$

and

$$W_j(\rho) = \frac{1}{\sigma_j^2} \int e^{i\rho \cdot K} F_{1j}(K)dK, \quad j = L,s. \tag{7.13}$$

With these definitions, (7.3) is clearly correct.

Since q_z is bounded by 2k, a sufficient condition for (7.4) is $(2k\sigma_s)^2 << 1$. And (6.37) gives conditions for the validity of (7.4), with σ, Γ replaced by σ_L, Γ_L.

Note that if $(q_z\sigma)^2 >> 1$, then for some k_* large enough, one can have both $(q_z\sigma_L)^2 >> 1$ and $(2k\sigma_s)^2 << 1$. However, if θ, θ' are such that $q_z \to 0$, then obviously (6.37) cannot be satisfied.

But if $(k\sigma)^2 >> 1$, then presumably it is possible to define a suitable k_* for θ, θ' bounded away from $\pi/2$, (so that q_z is bounded below).

And if $q_z \to 0$, the standard perturbation solution using the full surface spectrum $F_1(K)$ will apply.

One further remark. Depending on the smoothness properties of $F_{1s}(K)$ in the neighborhood of $K=q_\perp$, the integral in (7.5) or (7.8) could be expanded in an asympototic series in Γ_x^2, Γ_y^2.

That is, neglecting shadowing,

$$\int_{-\infty}^{+\infty} F_{1s}(q_z(\gamma-\gamma_0))w_e(\gamma;\alpha,\beta)d\gamma = F_{1s}(q_\perp)P(\gamma_0)+0(\Gamma_x^2+\Gamma_y^2) \tag{7.14}$$

Given a smooth enough spectrum around $q_\perp$, further terms in the expansion begun in (7.14) can easily be supplied, via Laplace's method.

So if $|q_\perp| > k_*$, and F_1 smooth near $q_\perp$, (7.5) can be approximated by function evaluations, eliminating the need to evaluate integrals. As $|q_\perp| \to k_*$, some attention has to be paid to the approaching cutoff in F_{1s}.

VIII. SCATTERING FROM A MOVING ROUGH SURFACE[27]

Now consider a surface varying in time, $z = \zeta(r,t)$. The basic assumption is that the variations in the surface $\zeta(r,t)$ and slopes $\gamma(r,t)$ are small relative to the incident frequency ω_0 and incident wave velocity $c = \omega_0/k$ respectively. Here the incident wave $U_0 \sim e^{-i\omega_0 t}$.

Such an assumption allows an immediate generalization of (6.14), namely

$$U(\alpha,\beta,t) = \frac{-ie^{ik(R_{10}+R_{20})-i\omega_0 t}}{4\pi R_{10}R_{20}} \int_{S_0} V(r)(q_z - q_\perp \cdot \gamma)$$

$$\exp\left(-i(q_\perp \cdot r + q_z \zeta(r,t)\right)dr \qquad (8.1)$$

As remarked several times before, such a formula will produce correct results for large or small Rayleigh parameters, and will be the only form considered in this section.

Again, the dominant contribution to the integral occurs near the stationary point, $\gamma_0 = -q_\perp/q_z$ and the term $q_z - q_\perp \cdot \gamma$ reduces to $|q|^2/q_z$ as before. Here the time variations of $\gamma(r,t)$ are assumed to be much slower than that of the exponential $\exp(-iq_z\zeta(r,t))$, so that the time dependence in γ may be neglected. For a small Rayleigh parameter, the term $q_z - q_\perp \cdot \gamma \cong |q|^2/q_z$ and may be taken outside the integral if $k\ell \gg 1$. This is because in this case, that of a gently sloping surface, the field is concentrated near the specular direction.

The function of interest in the case of a moving surface is the power spectrum of the field, defined by

$$S(\alpha,\beta;\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} < U(\alpha,\beta;t+\tau)U^{*}(\alpha,\beta;t) > e^{i\omega\tau}d\tau. \tag{8.2}$$

Introducing (8.1) into (8.2), using the approximations indicated, and proceeding as in Section VI in deriving (6.32), the result is

$$S(\alpha,\beta;\omega) = \frac{|V|^2|S_0||q|^4}{2\pi(4\pi R_1R_2q_z)^2}\int_{-\infty}^{+\infty} \exp\left(-iq_{\perp}\cdot\rho+i(\omega-\omega_0)\tau\right)$$

$$\cdot\ f_2(q_z,-q_z;\rho,\tau)d\rho d\tau \tag{8.3}$$

where $f_2(q_z,-q_z;\rho,\tau) = \exp\left(-q_z{}^2\sigma^2(1-W(\rho,\tau))\right)$ for a Gaussian surface.

The result in (8.3) could be shadowed by multiplying by $P(\alpha,\beta)$ as in earlier sections. This is consistent with the time dependence of ζ and γ, for the stationary assumption implies that the probability density function for ζ, and therefore for γ, is independent of time.

Suppose $(q_z\sigma) << 1$. Then expanding f_2 in powers of $(q_z\sigma)^2W$, the result is

$$S(\alpha,\beta;\omega) \sim \sum_{n=0}^{\infty} \frac{q_z{}^{2n}}{n!} S_n(\alpha,\beta;\omega) \tag{8.4}$$

where the term multiplying the sum is evident from (8.3), and

$$S_n(\alpha,\beta;\omega) = \int_{-\infty}^{+\infty} \exp(-iq_\perp \cdot \rho + i(\omega-\omega_0)\tau)(\sigma^2 W(\rho,\tau))^n d\rho d\tau \qquad (8.5)$$

But as the Fourier transform of $\sigma^2 W(\rho,\tau)$ is $F(K,\omega)$, as given in (1.11) or (1.15), (8.5) asserts that S_n is the n-fold convolution of F. The first few terms are as follows:

$$S_0(\alpha,\beta;\omega) = (2\pi)^3 \delta(q_\perp)\delta(\omega-\omega_0) \qquad (8.6)$$

$$S_1(\alpha,\beta;\omega) = F_1(q_\perp)\delta(\omega-\omega_0-\omega_+(q_\perp)) \qquad (8.7)$$

$$S_2(\alpha,\beta;\omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{+\infty} F_1(K)F_1(q_\perp - K)\delta\big(\omega_+(K)+\omega_+(q_\perp - K)-(\omega-\omega_0)\big)dK \qquad (8.8)$$

where $K = (k_x, k_y)$, as before.

S_0 represents the spike directed along the specular path, at frequency ω_0. For the moving ocean surface, if $F_1(K)$ is assumed zero in some circle (interval) around $K=0$, depending upon the wind speed above the ocean, then S_1 will also be zero in some interval around ω_0. S_2 is in general not zero for $\omega \cong \omega_0$, and will fill in the gap around the central spike left by S_1[28].

Now suppose $(q_z\sigma)^2 >> 1$. Then, as before, f_2 decays so rapidly for $W(\rho) \neq 1$, $(\rho \neq 0)$ that the integral in (8.3) may be approximated by replacing $1 - W(\rho,\tau)$ by a Taylor series in ρ and τ. The result is that S is Gaussian in nature,

$$S(\alpha,\beta;\omega) \cong \frac{1}{\Delta\omega\sqrt{\pi}} \exp\left(\frac{((\omega-\omega_0)-\omega_m)^2}{(\Delta\omega)^2}\right) \qquad (8.9)$$

where

$$\omega_m = \frac{D_1}{d}, \quad (\Delta\omega)^2 = -2q_z{}^2\sigma^2\frac{D}{d} \; . \tag{8.10}$$

Here D is the determinant of the Hessian matrix of W (with respect to ρ_x, ρ_y, and τ) evaluated at $\rho=0$, $\tau=0$, and d is the 2x2 principal minor of D. D_1 is D, with the first row of D replaced by $(q_\perp, 0)$. The second derivatives of W needed above can be calculated in terms of F_1 and ω_+, see (1.12) or (1.16).

If the scattering surface is not small, so that the r integral must be evaluated, the result is

$$S(R,\omega) = \frac{1}{|S_0|} \int_{S_0} S(\alpha,\beta;\omega)dr \tag{8.11}$$

Again, to obtain the correct normalization, particularly if S_0 is so large so as to include scattering near grazing incidence, (8.11) should probably include a shadowing correction.

Furthermore, if ζ is decomposed into the sum of a large and small surface, a composite surface form of (8.3) could be derived, which would combine the contributions to S from the large surface (using (8.9) evaluated with respect to ζ_L), and the contribution to S from the small surface, (using (8.4), evaluated in terms of ζ_S), perhaps convolved with (8.9). In particular for low grazing angles, the term $F_1(q_\perp)$ in (8.7) would be replaced by an integral as in (7.8).

APPENDIX A

To evaluate the integral

$$I = \iint \delta(f_1(x,y))\delta(f_2(x,y))\, dxdy,$$

introduce the new variables, $u = f_1(x,y)$, $v = f_2(x,y)$. Then $dxdy = J^{-1}dudv$, where J is the Jacobian determinant $\left|\frac{\partial(f_1 f_2)}{\partial(x,y)}\right|$. Then $I = \iint \delta(u)\delta(v)J^{-1}dudv = J^{-1}\Big|_{v=0}^{u=0}$; using the fact that the mapping $(x,y) \rightarrow (u,v)$ is invertible at $u=0=v$.

To apply this to the evaluation of the integral following (5.34) of the main text

$$I = \iint \frac{1}{R_1{}^2R_2{}^2}\delta\left(\frac{D-r}{R_2} - \frac{r}{R_1}\right)dxdy, \tag{A.1}$$

observe that

$$f_1(x,y) = \frac{x_1-x}{\sqrt{z^2+(x_1-x)^2+(y_1-y)^2}} - \frac{x}{\sqrt{z_0{}^2+x^2+y^2}}, \tag{A.2}$$

and

$$f_2(x,y) = \frac{y_1-y}{R_2} - \frac{y}{R_1}. \tag{A.3}$$

Now $f_1=0=f_2$ at $r=r_0$, where

$$r_0 = (x_0,y_0) = \frac{z_0}{z_0+z}(x_1,y_1) = D\frac{z_0}{z_0+z} \tag{A.4}$$

By a simple calculation,

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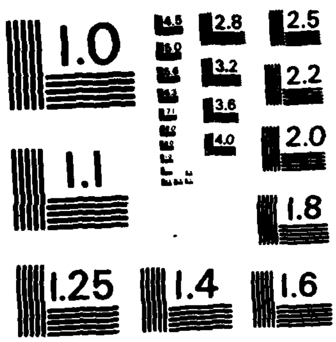

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$$R_1^{\,2}(r_0) = \frac{z_0^{\,2}}{(z_0+z)^2}\left(\left|D\right|^2 + (z_0+z)^2\right) = \frac{z_0^{\,2}B^2}{(z_0+z)^2} \qquad (A.5)$$

$$R_2^{\,2}(r_0) = \frac{z^2}{(z_0+z)^2}B^2 \qquad (A.6)$$

Here B is the distance between the image source and receiver, and is defined in A.5. Further, if J denotes the Jacobian determinant for f_1, f_2 given in A.2, A.3, then

$$J^{-1}\Big|_{r=r_0} = \frac{B^4z^2z_0^{\,2}}{(z_0+z)^6} \qquad (A.7)$$

But then

$$I = \frac{1}{R_1^{\,2}(r_0)R_2^{\,2}(r_0)}\, J^{-1}\Big|_{r=r_0} = \frac{1}{(z+z_0)^2} \qquad (A.8)$$

Note that $\alpha_0 = (\frac{r_0}{R_{10}}, \frac{-z_0}{R_{10}}) = (\alpha_{o\perp}, \alpha_{oz})$ and $\beta_0 = (\frac{D-r_0}{R_{20}}, \frac{z}{R_{20}}) = (\beta_{o\perp}, \beta_{oz})$. Further,

$$\alpha_{o\perp} = \beta_{0\perp} = \frac{r_0}{R_{10}} = \frac{D-r_0}{R_{20}} = \frac{D}{B}\,. \qquad (A.9)$$

In addition

$$\alpha_{oz} = \frac{-z_0}{R_{10}} = -\frac{(z_0+z)}{B} = -\cos\theta_0 \qquad (A.10)$$

and

$$\beta_{oz} = \frac{z}{R_{20}} = -\frac{z_0+z}{B} = \cos\theta_0. \qquad (A.11)$$

From (A.11) and noting, from A.5 and A.6 that $R_{10} + R_{20} = B$, A.8 becomes

$$I = \frac{1}{(z+z_0)^2} = \frac{1}{\beta_{oz}{}^2(R_{10}+R_{20})^2}, \qquad (A.12)$$

which is the desired result.

Using the relations in (A.9) and (A.10),

$$1-\alpha_{o\perp}\cdot\beta_{o\perp} = 1- \frac{|D|^2}{B^2} = \frac{(z_0+z)^2}{B^2} = \alpha_{oz}\cdot\beta_{oz} \qquad (A.13)$$

so that the rigid surface kernel and the free surface kernel agree at r_0.

APPENDIX B

The function $\tilde{F}(\sigma,\beta)$ has the following form:

$$2k^2\tilde{F}(\alpha,\beta) = \frac{k(\rho_2-\rho)\sqrt{1-|\beta_\perp|^2}}{\rho\sqrt{k_2{}^2-k^2|\beta_\perp|^2} + k\rho_2\sqrt{1-|\beta_\perp|^2}} \cdot$$

$$\left\{(1+V)\left[\frac{k^2\rho_2-k_2{}^2\rho}{\rho_2-\rho} - k^2\alpha_\perp\cdot\beta_\perp\right] - k\alpha_z(1-V)\sqrt{k_2{}^2-k^2|\beta_\perp|^2}\right\}.$$

Here k_2, ρ_2 refer to medium below the interface.

Appendix C

Representative parameter values for ocean surface using fully developed Phillips spectrum,

$$F_1(k) = B/k^3,\ g/u^2 \leq k \leq k_s \text{ (one dimension).}$$

Then

$$\sigma \cong u^2 \sqrt{\frac{B}{2g^2}} = .0049u^2 \text{ m.}$$

$$2\Gamma^2 \cong B \log \left(\frac{k_s u^2}{g}\right)$$

Here u = wind velocity, (m/sec), g = 9.81 m/sec^2, B = .0046 is a dimensionless constant, k_s is the cut-off between gravity and capillary waves, $k_s \cong 2\pi/.3\ m^{-1}$

Table 1

u(knots)	u(m/sec)	σ(m)	Γ	ℓ(m)
10	5.1	.13	.096	1.35
20	10.3	.52	.112	4.65
30	15.4	1.2	.120	10.0
40	20.6	2.1	.125	16.8

Let $k = 2\pi f/c$, f in hertz, $c \cong 1500$ m/sec.

Table 2
Values of $k\sigma$ ($k\ell$), as u and f vary.

f(Hertz)	u (Knots) 10	20	30	40
100	.055 (.57)	.22 (2.0)	.5 (4.2)	.88 (7.1)
200	.11 (1.1)	.44 (4.0)	1.0 (8.4)	1.8 (14.)
400	.22 (2.3)	.88 (8.0)	2.0 (17.)	3.6 (28.)
800	.44 (4.6)	1.8 (16.)	4.1 (34.)	7.1 (56.)
1600	.87 (9.2)	3.5 (32.)	8.0 (68.)	14.1 (110)

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